Li6 S.E.(EXTC) Sem III (R) 08/06/09

Applied Mathematics-III

(3 Hours) [Total Marks : 100

3 pm. to 6 pm.

N.B. (1) Question No. **1** is **compulsory**.
(2) Answer any **four** out of the remaining **six** qeustions.
(3) **Figures** to the **right** indicate **full** marks.

1. (a) Find the adjoint and inverse of $A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$. 5

(b) Obtain the Fourier series of 5

$$f(x) = \begin{cases} 0, & -\pi \le x \le 0 \\ x^2, & 0 \le x \le \pi \end{cases}$$

where f(x) is periodic with period 2π.

(c) State the first shifting theorem for Laplace Transforms. Use the theorem to obtain $L(e^{-t} \sin^2 t)$. 5

(d) Find the inverse Z transform of $(z-5)^{-3}$ when $|z| > 5$. 5

2. (a) Find the Fourier Transform of $f(x) = \begin{cases} 1, & |x| < k \\ 0, & |x| > k \end{cases}$ and hence evaluate $\int_{-\infty}^{\infty} \frac{\sin ks}{s} ds$. 7

(b) Represent $f(x) = \frac{\sin \pi x}{L}$, $0 < x < L$ by a half range consine series. 7

(c) Discuss the values of a and b, for which the following system of equations has — (i) no solution (ii) a unique solution and (iii) infinite number of solutions :— 6

$x + 2y + 3z = 6$
$x + 3y + 5z = 9$
$2x + 5y + az = b$.

3. (a) (i) Obtain $L(f(3t))$ if $L(f(t)) = \frac{20 - 4s}{s^2 - 4s + 20}$. 3

(ii) Given $L(\sin \sqrt{t}) = \frac{\sqrt{\pi}}{2s^{3/2}} e^{-(1/4s)}$, prove that $L\left(\frac{\cos \sqrt{t}}{\sqrt{t}}\right) = \sqrt{\frac{\pi}{s}}\, e^{-(1/4s)}$. 4

(b) Find non-singular matrices P and Q such that the normal form of 7

$$A = \begin{bmatrix} 1 & 3 & 6 & -1 \\ 1 & 4 & 5 & 1 \\ 1 & 5 & 4 & 3 \end{bmatrix}$$ is PAQ. What is the rank of A ?

(c) Find the Fourier series expansion of $f(x) = x^3$ in $(-\pi, \pi)$. 6

4. (a) State the convolution theorem for inverse Laplace transforms. Hence or otherwise obtain $L^{-1}\left(\frac{16}{(s-2)(s+2)^2}\right)$. 7

(b) If the Fourier sine transform of f(x) is $\frac{e^{-as}}{s}$, find f(x). Hence obtain the inverse Fourier sine transform of $\frac{1}{s}$. 7

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(c) Solve by the Gauss elimination method :— 6

$$x + 4y - z = -5$$
$$x + y - 6z = -12$$
$$3x - y - z = 4.$$

5. (a) Obtain the Fourier series expansion for the function $f(x) = x^2$ in $(0, a)$. Hence deduce that $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots$ 7

(b) (i) Find $L^{-1}\left(\frac{a(s^2 - 2a^2)}{s^4 + 4a^4}\right)$. 4

(ii) Evaluate $L^{-1}\left(\int_s^{\infty}\left(\frac{u}{u^2 + a^2} - \frac{u}{u^2 + b^2}\right) du\right)$. 3

(c) Find $Z(k^2 e^{-ak})$, $k \geq 0$. 6

6. (a) (i) Using Laplace transform, show that $\int_0^{\infty} e^{-t}\left(\frac{\cos at - \cos bt}{t}\right) dt = \frac{1}{2}\log\left(\frac{b^2 + 1}{a^2 + 1}\right)$. 7

(ii) Given $L\left(\operatorname{erf}\sqrt{t}\right) = \frac{1}{s\sqrt{s+1}}$, evaluate $\int_0^{\infty} te^{-t^2} \operatorname{erf}(t)\, dt$.

(b) Find Z-transform and the Radius of convergence for the following sequences : 7

(i) $f(k) = \frac{5^k}{k!}$, $k \geq 0$

(ii) $f(k) = 2^k$ $k < 0$.

(c) When do you say that vectors $X_1, X_2, \dots, X_n$ are linearly dependent ? Are the vectors $X_1 = [\,3 \;\; 1 \;\; 1\,]$, $X_2 = [\,2 \;\; 0 \;\; -1\,]$, $X_3 = [\,4 \;\; 2 \;\; 1\,]$ linearly dependent ? 6

7. (a) Obtain the complex form of the Fourier series for $\cos ax$, where a is not an integer, in $(-\pi, \pi)$. 7

(b) Solve using Laplace transform :— 7

$$\frac{d^2y}{dt^2} + y = t,\quad y(0) = 1,\ y'(0) = 0.$$

(c) Solve by the Gauss-Seidel method : (Go up to 4 iterations) 6

$$28x + 4y - z = 32$$
$$2x + 17y + 4z = 35$$
$$x + 3y + 10z = 24$$