S.E. (I.T.) Sem III (R) 08/06/09

Con. 2642-09. (REVISED COURSE) VR-3390

Lib

(3 Hours) [Total Marks : 100

Applied Mathematic - III

3 p.m. to 6 p.m.

N.B. : (1) Question No. **1** is **compulsory**.

(2) Attempt any **four** questions out of remaining **six** questions.

(3) Make **suitable** assumptions if **required** and **justify** the **same**.

(4) **Figures** to the **right** indicate **full** marks.

1. (a) Show that every square matrix can be uniquely expressed as the sum of a symmetric matrix and skew symmetric matrix. 5

(b) Obtain the complex form of Fourier Series for $f(x) = e^{-x}$ in $(-1, 1)$. 5

(c) Find the Laplace transforms of the following : 5

(i) $\sqrt[t]{1 + \sin t}$ (ii) $te^{3t} \sin t$.

(d) Construct the analytic function whose real part is $e^{-x}(x \cos y - y \sin y)$. 5

2. (a) Prove that $\int_0^\infty e^{-t} \frac{\sin^2 t}{t}\, dt = \frac{1}{4} \log 5$. 6

(b) Find the Fourier expansion of $\cos px$ in $(0, 2\pi)$. Hence, deduce that 8

$$\pi \operatorname{cosec} \pi x = \frac{1}{p} + \sum_{n=1}^{\infty} (-1)^n \left[\frac{1}{p+n} + \frac{1}{p-n}\right].$$

(c) Show that $u = \cos x \cosh y$ is a harmonic function. Find its harmonic conjugate and the corresponding analytic function. 6

3. (a) Find the analytic function $f(z) = u + iv$ in terms of z if $u + v = \frac{x}{x^2 + y^2}$. 6

(b) Obtain Fourier series for 8

$$f(x) = x + \frac{\pi}{2}, \quad -\pi < x < 0$$

$$= x + \frac{\pi}{2}, \quad 0 < x < \pi$$

Hence, deduce that $\frac{\pi^2}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots\dots$

(c) Find the orthogonal trajectory of the family of curves given by $2x - x^3 + 3xy^2 = a$. 6

4. (a) Find the Laplace transform of the following – 6

(i) $\int_0^t u \cos^2 u\, du$ (ii) $te^{3t} \operatorname{erf} \sqrt{t}$.

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(b) Reduce A to normal form and find its rank where **6**

$$A = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 4 & 6 & 8 & 10 \\ 15 & 27 & 39 & 51 \\ 6 & 12 & 18 & 24 \end{bmatrix}$$

(c) Obtain half rangs sine and cosine series for $f(x) = lx - x^2,\ 0 < x < l$. **8**

5. (a) Find inverse Laplace transform of the following : **8**

(i) $\dfrac{s^2 + 2s + 3}{(s^2 + 2s + 5)(s^2 + 2s + 2)}$ (ii) $\tan^{-1}\left(\dfrac{s + a}{b}\right)$.

(b) Use the adjoint method to find the inverse of A where **6**

$$\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$$

(c) If $f(a) = \displaystyle\int_C \frac{4z^2 + z + 5}{z - a}\,dz$, where C is $|z| = 2$, find the values of **6**

$f(1),\ f(i),\ f'(-1),\ f''(-i)$.

6. (a) Solve using Laplace transform $\dfrac{d^2 y}{dt^2} + 9y = 18t$ given that $y(0) = 0$ & $y(\pi/2) = 0$. **8**

(b) Determine l, m, n and find A^{-1} if $A = \begin{bmatrix} 0 & 2m & n \\ l & m & -n \\ l & -m & n \end{bmatrix}$ is orthogonal. **6**

(c) Evaluate $\displaystyle\int_C \frac{1}{z^4(z + 4)}\,dz$, where C is $|z| = 2$. **6**

7. (a) Expand $f(z) = \dfrac{1}{z^2(z - 1)(z + 2)}$ about $z = 0$ **8**

(i) $|z| < 1$, (ii) $1 < |z| < 2$, (iii) $|z| > 2$.

(b) Solve the equations $2x + y - z = 1,\ \ x - 2y + 3z = 6,\ \ x - y + 2z = 9$. **6**

(c) Find the inverse Laplace transform of the following – **6**

(i) $\dfrac{e^{4-3s}}{(s + 4)^{5/2}}$, (ii) $\dfrac{8e^{-3s}}{s^2 + 4}$.
