Lib T.E. (ETRX) Sem VI (R) 2/06/09 3 p.m. to 6 p.m.

Con. 2898-09. Discrete Time Signal Processing **VR-5313**

(3 Hours) **[Total Marks : 100**

N.B.: (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions out of remaining **six** questions.
(3) Assume **suitable data** if **necessary**.

1. (a) Determine the output y(n) of a relaxed LTI system with impulse responses $h(n) = a^n u(n)$, $|a| < 1$ when the input is a unit step sequence is : $x(n) = u(n)$. **20**

(b) Determine the Z-transform of the signal $x(n) = n\, a^n u(n)$.

(c) Indicate whether the following systems are minimum phase, maximum phase or mixed phase by finding their zeros.

$$H_1(z) = 6 + z^{-1} - z^{-2}$$

$$H_2(z) = 1 - z^{-1} - 6z^{-2}$$

$$H_3(z) = 1 - \frac{5}{2} z^{-1} - \frac{3}{2} z^{-2}$$

$$H_4(z) = 1 + \frac{5}{3} z^{-1} - \frac{2}{3} z^{-2}$$

$$H_5(z) = 3 + \frac{1}{2} z^{-1} - \frac{1}{2} z^{-2}$$

(d) State and explain Parseval's Theorem in DFT. How can it be used to find the energy of a finite duration sequence.

2. (a) Obtain autocorrelation of the following sequence : **4**

$x_1(n) = [3, 1, -2, 1]$ (↑)

(b) If $x_1(n)$ in part (a) is delayed in time and is given by $x_2(n) = x_1(n-2)$, find the correlation between them. What do you conclude from the result ? **5**

(c) Prove that a discrete sinusoid is periodic only if the discrete frequency is a rational number. **5**

(d) Test Linearity and Time invariance of the following systems. **6**

(i) $y(n) = |x(n)|$
(ii) $y(n) = (n+1)\, x(n)$
(iii) $y(n) = e^{x(n)}$

3. (a) Determine the causal signal x(n) whose Z-transform is given by— **8**

$$X(z) = \frac{1}{\left(1+z^{-1}\right)\left(1-z^{-1}\right)^2}$$

(b) Determine and sketch energy density spectra of the signals. **8**

(i) $x(n) = \left(\frac{1}{2}\right)^n u(n)$ (ii) $x(n) = \left(-\frac{1}{2}\right)^n u(n)$

(c) The transfer function of a system is given by $H(z) = \frac{1-z^{-4}}{1-z^{-1}}$. Is the system stable ? Justify giving pole zero diagram and ROC. **4**

[TURN OVER

4. (a) Find the magnitude response for a filler having transform function 6

$$H(z) = \frac{z-4}{z-\frac{1}{4}}.$$

(b) A certain discrete time system has the following poles and zeros. 8

$$z_1 = 0, \quad z_2 = -\frac{1}{3}, \quad \frac{z}{3} = -\frac{1}{6}$$

$$p_1 = \frac{1}{2}, \; p_2 = p_3 = 0{\cdot}8\, e^{\pm j\pi/3}$$

If the magnitude response $\left| H\left(e^{jw}\right) \right|_{w=0} = \frac{10}{9}$

Find the gain and complete transfer function of the system.

(c) Find the values x(0) and x(∞) for a function x(n) whose transfer function is given by) 6

$$X(z) = \frac{1}{1-0{\cdot}1z^{-1}-0{\cdot}2z^{-2}}$$

5. (a) A system is defined by the following differnce equation 8

$$y(n) - \frac{1}{6}\, y(n-1) - \frac{1}{6}\, y(n-2) = x(n)$$

(i) Realize the system using Direct form II and parallel realisation.

(ii) Comment on the stability of the system.

(b) A certain discrete time filter has poles located at 0·6 and 0·2. The filter uses a four bit processor in which MSB represents the sign bit and the remaining three bits store quantised coefficients. 12

(i) What is the effect of Quantization on pole locations if direct form II is used for realization ?

(ii) If cascade form is used for realization, what is the change in pole values after quantization ?

(iii) In which case is the effect of Quantization less ?

6. (a) Determine the output response of a system if input x(n) and impulse response h(n) are given as 8

$$x(n) = \{ \underset{\uparrow}{1},\ 2,\ 3,\ 3,\ 1 \}$$

$$h(n) = \{ \underset{\uparrow}{2},\ 1,\ 2 \}$$

Use circular convolution only.

(b) A four point complex sequence x(n) is given as 12

$$x(n) = \{ 1 + j2,\ 1 + j,\ 2 + j,\ 2 + j2 \}$$

(i) Find X(k)

(ii) Using the result from (1), find DFTs of the sequences $x_1(n)$ and $x_2(n)$ given below :

$x_1(n) = \{ 1, 1, 2, 2 \}$

$x_2(n) = \{ 2, 1, 1, 2 \}$.

7. (a) Find DFT of the following sequence using DIT FFT. 8

$$x(n) = \{ 1, 1, 1, 1, 1, 1, 0, 0 \}$$

(b) Using the results derived in (a) find DFT of the signal 2

$$y(n) = \{ 1, 0, 0, 1, 1, 1, 1, 1 \}$$

(c) Explain the architecture of DSP processor. 5

(d) Find IDFT of the given sequence using IFFT ? 5

$$X(k) = \{ 10, -2 + j2, -2, -2 - j2 \}$$