FE Sem-I (R) - App Chem-I 9/May 2012

Revised Course

GN- 9869

2 Hours

Total marks 75

N. B. (i) Question No 1 is **compulsory**.
(ii) Attempt **any four** from remaining **six**.
(iii) Figures to the **right** indicate full marks.
(iv) All questions carry equal marks.
(v) Atomic weights: H=1, C=12, N=14, O=16, Na=23, Mg=24, S=32, Cl=35.5, Ca=40.

Q.1. Attempt any **Five** from the following **15**

a. Why is the Hot lime Soda process preferred over the Cold lime soda process?
b. What are nano materials? Mention two reasons why properties of materials differ at the nano scale.
c. Distinguish between conventional and non-conventional energy sources.
d. What is the repeat unit structure of natural rubber? State any two drawbacks of natural rubber.
e. What are plain carbon steels? What are their drawbacks?
f. Find the acid value of a used lubricating oil sample whose 10 ml required 5 ml of N/50 KOH during titration. (Density of oil=0.91 g/cc). State whether the oil is suitable for lubrication or not.
g. Explain the principle involved in production of hydropower. List any two advantages and disadvantages of hydropower generation.

Q.2

a. Explain with a neat diagram, the Zeolite process of water softening including the following points
(i) principle (ii) process (iii) softening and regeneration reactions
(iv) advantages & limitations **6**
b. Explain structure, properties and uses of fullerenes **5**
c. 2.5 g of a blended oil was saponified using excess alcoholic KOH solution (0.5N). After refluxing for two hours, the mixture was titrated against 0.5N HCl solution. The burette reading was found to be 24 ml. The blank titration required 40 ml of the same HCl solution. Find the saponification value of the oil. If the oil used for blending has saponification value of 191, calculate percentage oil in the blend. **4**

Q.3

a. Calculate lime (95% pure) and soda (90% pure) required for softening one million litres of water containing the following constituents:
$Ca(HCO_3)_2$=81mg/l, $Mg(HCO_3)_2$=73mg/l, $CaSO_4$=68mg/l, $MgCl_2$=95mg/l, $Mg(NO_3)_2$=14.8mg/l, H_2SO_4= 14.7 mg/l **6**
b. In what situations are solid lubricants used? Explain structure, properties and uses of any **one** solid lubricant. **5**
c. What are the functions of the following constituents in the compounding of plastics
(i) Fillers (ii) Plasticizers **4**

Q. 4

a. Write preparation properties and uses of any **two** of the following **6**
(i) Polymethyl methacrylate (PMMA)
(ii) Urea formaldehyde
(iii) Buna-S

[Turn over

b. An exhausted Zeolite softener was regenerated by passing 300 litres of NaCl solution having a strength of 150 g per litre of NaCl. If the hardness of water sample was 480 ppm, calculate the total volume of water softened by this softener. **5**

c. Explain special effects of the following metals on properties of alloy steels **4**

i. Chromium (ii) Nickel (iii) Cobalt (iv) Tungsten

Q. 5.

a. What are carbon nanotubes? Describe production of single walled carbon nano tube by LASER method. **6**

b. Using phase rule, find the number of degrees of freedom (F) in the following systems at equilibrium. **5**

i. In the water system, when

Ice (s) $\rightleftharpoons$ water (l) $\rightleftharpoons$ water vapour (g)

ii. A gaseous mixture of Nitrogen and Hydrogen

c. Write a note on nickel-hydrogen batteries. **4**

Q.6.

a. What is meant by fabrication of plastics? Explain Compression moulding with the help of a neat diagram. **6**

b. Explain application of Gibbs Phase Rule to one component system- water system. **5**

c. Describe the working of solar heating system using Flat Plate collector. **4**

Q. 7.

a. Write a short note on any **one** of the following **5**

i. Haeckelites (ii) Hydrogen as fuel (iii) Reverse Osmosis

b. Explain Boundary lubrication. **5**

c. Define glass transition temperature of polymer. What factors influence its value. **5**

FE/Sem-I (ReN) 5/6/2012
Applied Physics-I

GN-9269

(2 Hours) [Total Marks: 75]

N.B. :- (1) Question **No.1** is compulsory.
(2) Attempt any **four** Questions from remaining questions.
(3) Figures to the right indicate full marks.

Que.No. 1. Attempt any FIVE.

(a) What is difference between sound intensity and loudness ? (3)
(b) Draw the following with respect to cubical unit cell (3)
(1 2 3) , (2 0 0) , [2 3 0]
(c) Define superconductors ,critical temperature , critical magnetic field. (3)
(d) Explain different phases of liquid crystal. (3)
(e) Explain use of CRO to determine phase difference between two signals. (3)
(f) Explain cavitation effect and its 'applications. (3)
(g) Explain characteristic X – ray spectra. (3)

Que.No. 2 (a) Define Piezoelectric effect . Explain construction and working of piezoelectric Oscillator. (8)
(b) Explain the concept of Fermi level .What is the probability of an electron being Thermally excited to conduction band in silicon at 20 ^{o}c .the band gap energy is 1.12ev Given Boltzman constant 1.38 10^{23} J/K (7)

Que.No. 3 (a) Find the following parameters for HCP structure (i) co-ordination No. (ii) Atomic Radius (iii)No. of atoms pre unit cell (iv)APF (v)c/a ratio. (8)
(b) State Bathe law. Explain electrostatic focusing. (7)

Que No. 4 (a) Derive Bragg's equation .Explain construction and working of Bragg's Spectrometer. (8)
(b) An electrically deflected CRT has a anode voltage 2000v and parallel deflecting Plates 1.5 cm long and 20 mm apart. If the screen is 50 cm from the center of deflecting Plates .Find (i) Beam speed (ii) the deflection sensitivity of the tube and (iii) Deflection Factor of the tube. (7)

Que No. 5 (a)Define Reverberation time. Explain how the reverberation time is affected by (i)size (ii)Nature of it's wall surface (iii) Audience (8)

(b) What is eco sounding techniques ? Velocity of ultrasonic in mild steel is 5.9×10^3 m/s Velocity of ultrasonic in brass measured by an ultrasonic gauge meter ,calibrated for Mild steel is 4.3×10^3 m/s .If thickness of brass plate measured by the gauge meter is 15 cm . What is the real thickness? (7)

Que. No. 6 (a) What is Meissner effect ? Explain Type I and Type II superconductors (8)

(b) The radiation of an x –ray tube operated at 50 KV are diffracted by a cubical KCl FCC Crystal of molecular wt. 74.6 and density 1.99 gm/cc .Calculate (i) The shortest wave length (ii)Glancing angle for first order reflection. (7)

Que.No. 7 (a) What is Hall effect ? State it's significance. How can mobility be determined by using Hall Effect? (8)

(b) Calculate P. F. for Chromium having BCC structure. Given density 5.98 gm/cc, atomic weight 50. (7)

FE / Sem-I (Rev) 31/5/2012
Computer Programming - I

GN-8669

(3 hours) **[Total: 100 Marks**

NB:

1. Question number **one** (01) is compulsory.
2. Attempt any **four** (04) questions from remaining.
3. Make suitable assumptions if required and state them clearly

Q.1 a.) **10**

I. Consider following code

```
void show ( int a ,int  b , int  c )
{ cout<< a<< "  " << b <<"  "<< c;
        }
int *I , j;
   i = &j;
    j = 2;
int k[ ] = { 1,2,3};
```

what will be output of following statements

```
show ( *I , j , *k );
show ( *k ,*k++ );
```

II. Select the correct option : A class becomes an abstract class when

i) Key word abstract precedes the class name
ii) All the functions in the class are pure virtual
iii) Atleast one function in the class ispure virtual
iv) It derives virtually from base class

III.

```
#define m ( a, b) ( a>b  ?  a: b )
void main()
{
int I = 5 , j = 15 ;
cout<< m( ++i , j++);
}
```

IV. What is the output of following code :

```
void swap ( int i , int j)
{int temp;
temp = i ; I = j ; j = temp;
}
void main( )
{
int k = 13 , m = 45;
swap ( k , m);
cout<< "k = " << k << "m = " << m;
}
```

V. Select the correct answer

while (expression)

{}

To terminate this loop value of expression should evaluate to

i) True

ii) False

b) Discuss the merits and demerits of Object Oriented Methodology. **10**

Q.2 a) Write a program to find number of and sum of all integers greater than 200 and less than 350 that are divisible by 9. **10**

b) Write a program which reads an integer N followed by N numbers; It prints each number and sum of the numbers read so far. **10**

Q.3 a) Differentiate between Structures and Unions from the point of view of Operations on them and memory allocation. **10**

b) Write a program to represent and process Complex Numbers as a structure. Process should include addition of complex numbers and printing them in x+iy format. **10**

Q.4 a) Write a method called ***delete(String str , int m)*** in an appropriate class that returns the input string with m th character removed. **10**

b) Explain various storage classes in c++. **10**

Q.5 a) Class String has a data member ***str*** of type char*. Distinguish from the point of view of constructors following statements: **10**

String name1 (" XYZ");

String name2(name1);

b) What are destructors? State the necessity of destructor in Q.5a. **10**

Q.6 a) Write a C++ program to add two matrices. **10**

b) Explain binary operator overloading using a suitable example. **10**

Q.7 Write notes on following (any two) **20**

a) Run time polymorphism

b) Data Abstraction and Data Encapsulation

c) Friend function

FE | Sem - I (Rev.) 18/6/12
BEE

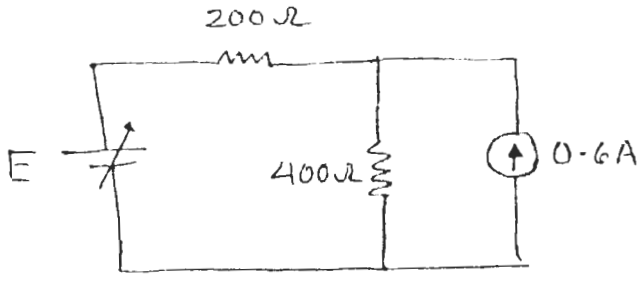
GN-1048

(3 hours) [Total Marks: 100

N.B. 1) Question No 1 is compulsory and solve any four questions from remaining six.
2) Assume suitable data if required and mention that assumption while solving that question.
3) Figures to the right indicate full marks.

Q1 Any Four. (20)

A) To what voltage should adjustable source E be set in order to produce a current of 0.3 A in 400 ohms resistor.

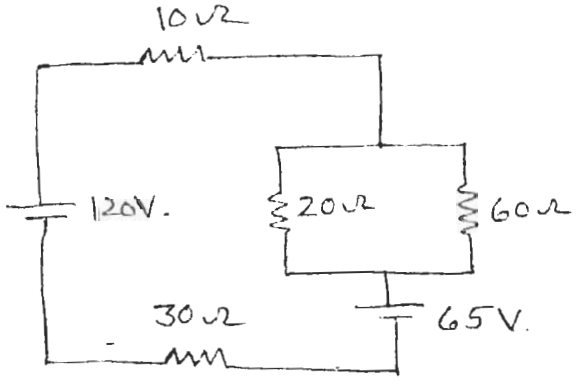


B) A series RLC circuit is connected to the ac voltage source $v(t) = 24\sin(1000t + 20^{\circ})$ V. If R = 15 ohms and L = 0.1 H, what value of capacitance C is required to make the phase angle between v(t) and i(t) in the circuit equal to zero degrees.

C) Explain two wattmeter method for measurement of 3 – phase power.

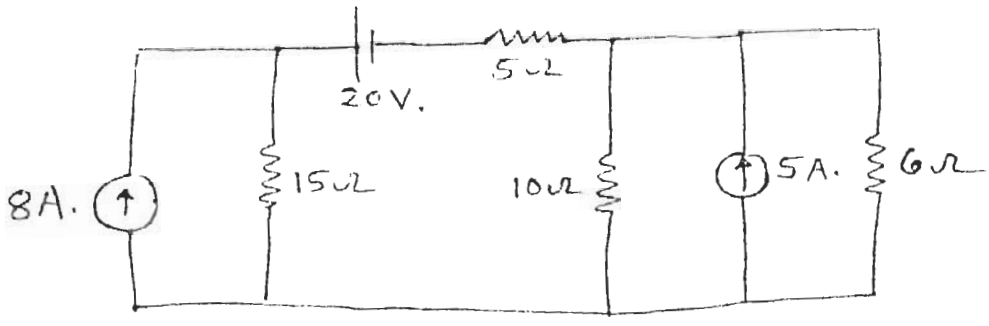
D) A 50 KVA,2200/440 V, 50 Hz single phase transformer has primary turns of 200.Determine 1)flux in core 2) secondary turns 3) rated primary current 4) rated secondary current.

E) Find the current which flows through 20 ohms resistor.

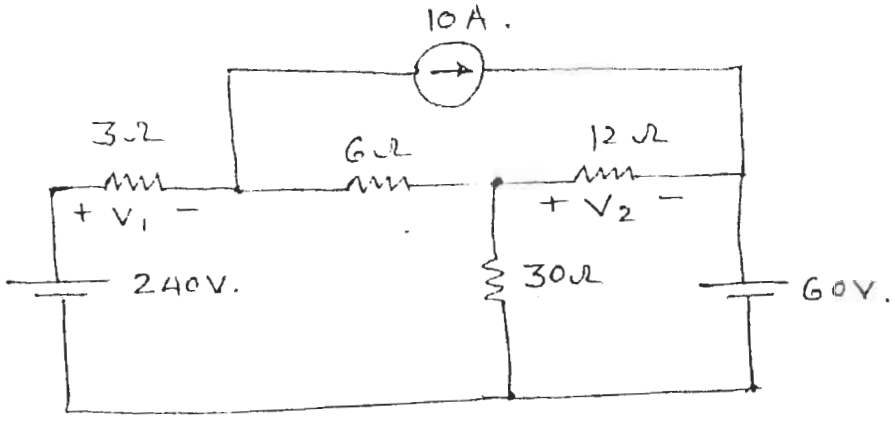


Q2 A) Find the current through 6 ohms resistor using Thevenin's Theorem for the given circuit. (12)

Verify the same using Superposition Theorem.



B) Use Nodal Analysis to determine 1) V_1 and V_2 2) Power absorbed by 6 ohms resistor. (08)



Q3 A) Two practical coils A and B are connected in series and excited by single phase ac supply (10) of 240 V, 50 Hz. Input from the supply to the circuit is 3 KW and 2 KVAR. If resistance of coil A is 5 ohms and inductance of coil B is 15 mH then calculate

1. Inductance of coil A
2. Resistance of coil B
3. Voltages across both the coils.

B) Three impedances are connected in parallel across 60Sin(wt) voltage source Vs. (10) Z1 = 50 ohms, Z2 = j40 ohms and Z3 = -j80 ohms. Find currents in each impedance, show that total current = voltage*total admittance. Also draw phasor diagram showing Vs , i_1, i_2, i_3, i_T.

Q4 A) 33 ohms resistance, 10 mH inductor and 0.1 micro farads capacitor is connected in series and excited by a supply voltage Vs = 13.2+j0 . V Find (10)

1. The resonant frequency in Hertz
2. Value of quality factor Q.
3. Polar forms of current and voltages across R, L and C at resonance.
4. Draw a phasor diagram showing all quantities in 3.

B) In a 50 KVA, 1100/220 V transformer, the iron and copper losses at full load are 350 W and 425 W respectively. Calculate the efficiency at (10)

1. Full load with unity power factor
2. Half load with unity power factor.
3. Full load with 0.8 pf lagging.

Also determine the maximum efficiency and the load at which maximum efficiency occurs assuming the load to be resistive.

Q5 A) A 5 KVA, 400/200 V ,50 Hz, single phase transformer gave the following results during open and short circuit tests, (10)

O.C. Test : 400 V , 1 A , 60 W ..... (H.V.Side)

S.C. Test : 15 V , 12.5 A, 50 W ..... (H.V.Side)

Calculate

1. No load parameters R_o and X_o.
2. Equivalent resistance and reactance referred to high voltage side.
3. Regulation at full load and 0.8 pf lagging.
4. Iron and copper losses at full load.
5. Efficiency at half load and 0.8 pf lagging.

B) Three inductive coils each with a 15 ohms resistance and 0.03 H of inductance are connected a) in star and b) in delta, to 3 – phase 400 V , 50 Hz supply. Calculate for each of the case (10)

1. Phase and line currents.
2. Total power absorbed.

Q6 A) Draw experimental set-up to plot Input/Output characteristics of CE BJT amplifier. Explain the characteristics and draw it. (08)

B) Draw and explain construction and working of D.C shunt Motor. Explain its characteristics and applications. (10)

C) Define RMS value in alternating waveforms. (02)

Q7 A) How is rotating magnetic field is produced in 3- phase Induction Motor. What is a slip in Induction Motor. (10)

B) Explain full wave bridge rectifier with resistive load. Find the expression for average voltage and rectifier efficiency. What is ripple factor. (10)

Con. 3667-12.

FE | sem - I (REV) 26/5/12
BEE

GN-4907

(3 Hours) [Total Marks : 100

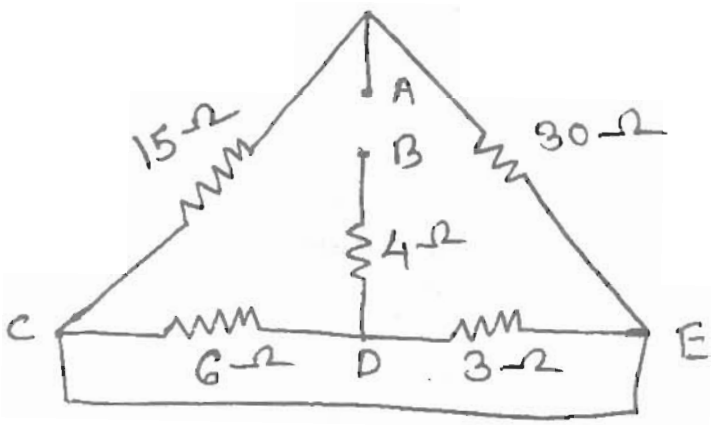
N.B. (1) Question No. **1** is **compulsory.**
(2) Attempt any **four** questions out of remaining **six** questions.

1. (a) Derive the relationship between Phase and Line voltages and currents for a star connected balanced load across a three-phase balanced system. **5**

(b) Four wires p, q, r and s are connected to a common point. The currents in the lines p, q and r are $6 \sin\left(wt + \frac{\pi}{3}\right)$, $5 \cos\left(wt + \frac{\pi}{3}\right)$ and $3 \cos\left(wt + \frac{\pi}{3}\right)$. Find the current in wire s. **5**

(c) Differentiate between Ideal and Practical transformers. **5**

(d) Calculate the effective resistance R_{AB} of given network. **5**



2. (a) Explain the input and the output characteristics of transistor in common emitter configuration. **10**

(b) A circuit has L = 0·2 H and inductive resistance 20 Ω is connected in parallel with 200 μF capacitor with variable frequency, 230 V supply. Find the resonant frequency at which the total current taken from the supply is in phase with supply voltage. Also find the value of this current. Draw the phasor diagram. **10**

3. (a) Each phase of three phase delta connected load has an impedance of $Z_{ph} = (50 \angle 60)\Omega$. The line voltage is 400 V. Calculate the power consumed by each phase and the total power. What will be the readings of the two wattmeters connected to measure the power ? **10**

(b) Draw the circuit diagram for bridge rectifier. Explain it and deriv equations for I_{dc}, I_{rms}, V_{dc} and η. **10**

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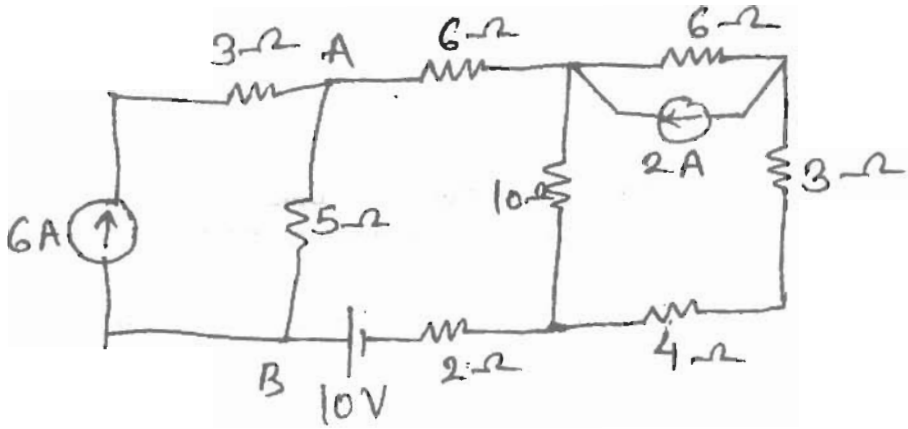
4. (a) Explain how rotating magnetic field is produced in 3-phase induction motor. **10**
   (b) By Norton's Theorem, find the current in 5Ω resistor. **10**



5. (a) Open circuit and short circuit tests on 5 kVA, 200/400 V, 50 Hz, single-phase transformer gave the following test results :— **10**

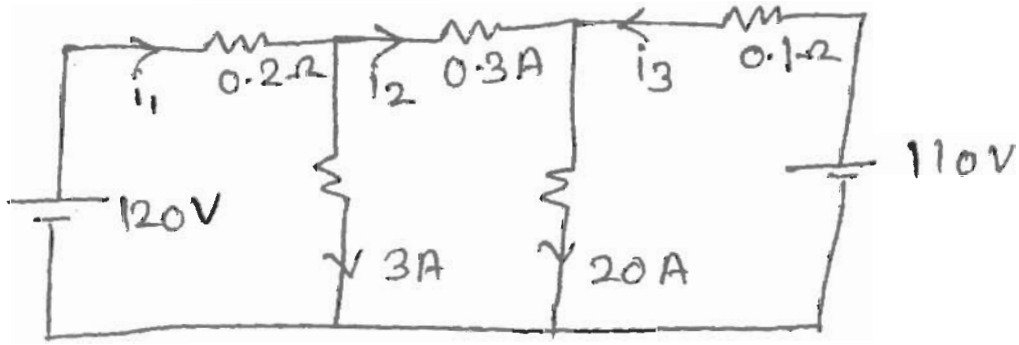
OC test	200 V	1 A	100 W
SC test	15 V	10 A	85 W

   (i) Draw the equivalent circuit referred to prim.
   (ii) Calculate the % η at 80% load, 0·8 p.f.

   (b) Explain how two wattmeters can be used to measure power and power factor in a 3ϕ balanced star connected load with lagging pf. **10**

6. (a) A coil of 0·6 p.f. is in series with a 100 μF capacitor and is connected to a 50 Hz supply. The potential difference across the coil is equal to the potential difference across the capacitor. Find the inductance and resistance of the coil. **10**
   (b) Three identical coils, each having a resistance of 10 Ω and an inductive reactance of 10 Ω are connected in delta, across 400 V, 3 ϕ supply. Find the line current and readings on each two wattmeter connected to measure the power. **10**

7. (a) By Nodal Analysis, find I_1, I_2, I_3. **10**



   (b) Write short notes on following :— **10**
   (i) Construction, classification and application of DC motor.
   (ii) Maximum Power Transfer Theorem.

FE | Sem.-I (Rev) 22/5/2017
Engg. Mechanics

Con. 3826-12.

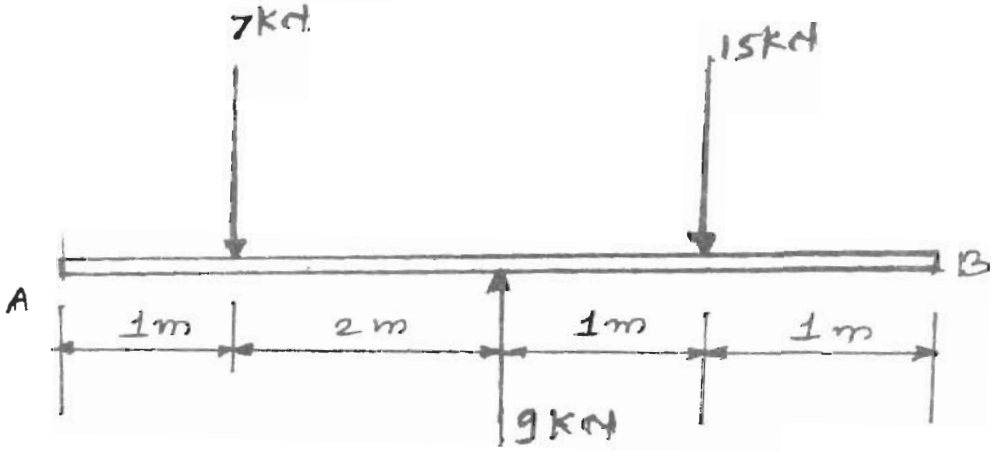
GN-4889

(3 Hours) **[Total Marks : 100**

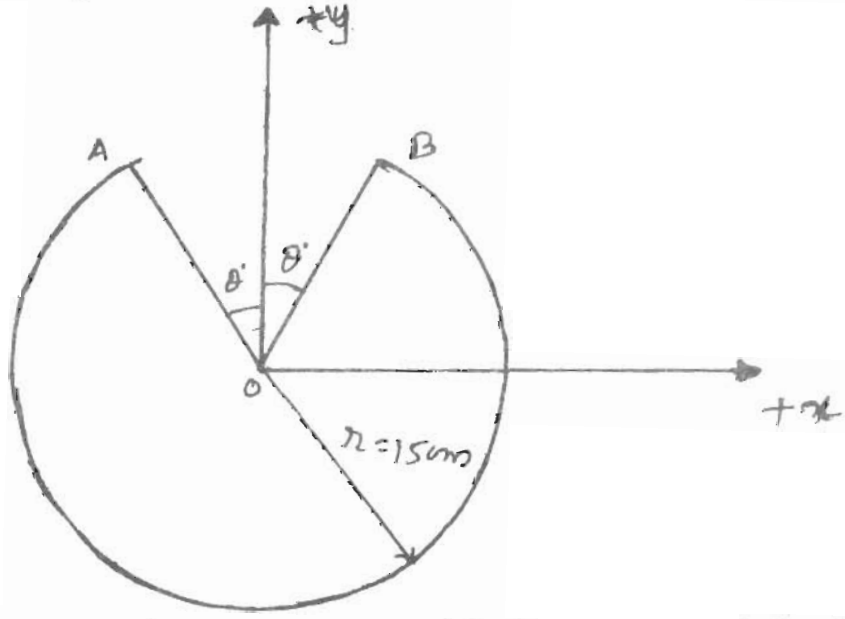
N.B. : (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions out of remaining **six** questions.
(3) Assume **suitable** data if **necessary** and mention the **same** clearly.
(4) Take g = 9·81 m/s².
(5) Draw **suitable** sketches where **necessary**.

1 Answer any **four** questions :— **5**

(a) The resultant of the three forces shown in **figure** and other two forces P and Q acting at 'A' and 'B' is a couple of magnitude 120 kNm clockwise. Determine the force P and Q.



(b) A thin homogeneous **wire** of uniform section is built into a shape as shown in **figure.** Determine the position of c.g. of a wire. Take θ = 30° and r = 15 cm. **5**



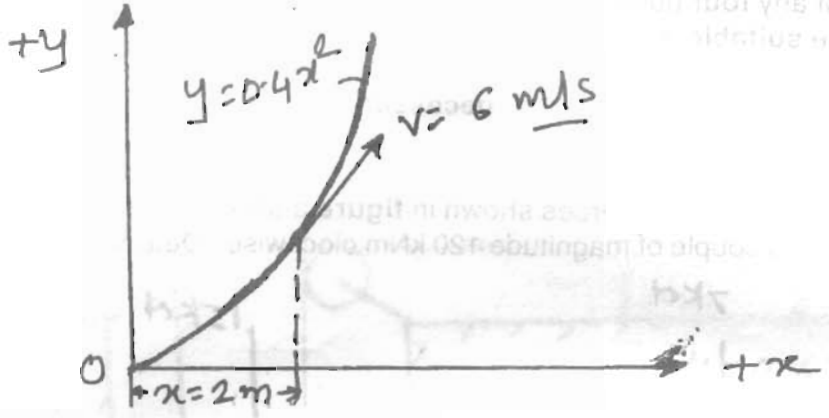
(c) During a test, the car, moves in a straight line such that its velocity is defined by **v** = 0·3 (9t² + 2t) m/s, where 't' is in seconds. Determine the position and acceleration when t = 3 sec. Take at t = 0, x = 0. **5**

(d) A man in a balloon is rising with a constant velocity of 5 m/s propels a ball upwards with a velocity of 2 m/s relative to the balloon. After what time interval will the ball return to the balloon. **5**

[TURN OVER

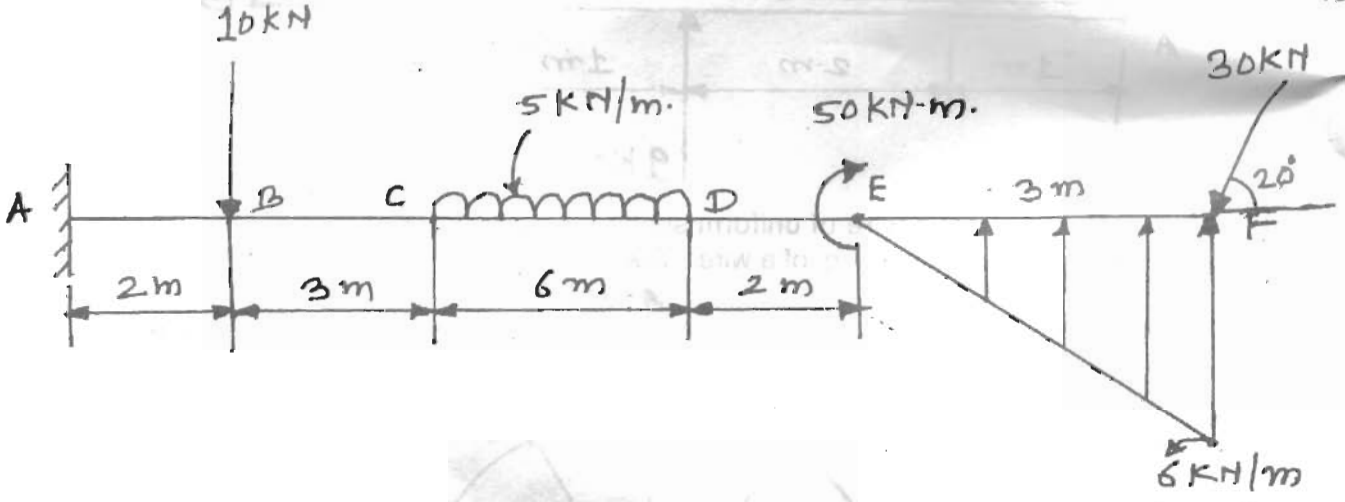
(e) A point moves along a curved path $y = 0{\cdot}4\,x^2$, At x = 2m its speed is 6 m/s increasing at 3 m/s^2. At this instant find — 5

(i) Velocity components along x and y directions. (ii) Its acceleration.

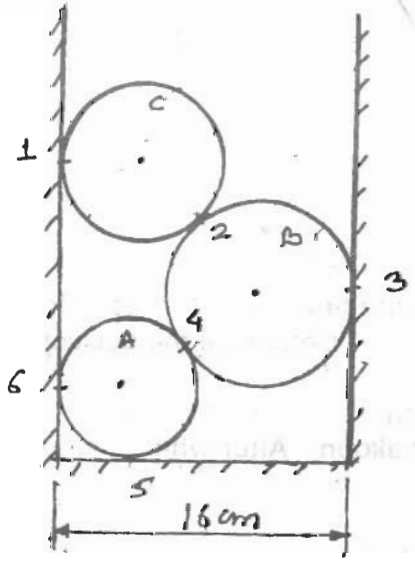


(f) State and prove work Energy principle. 5

2. (a) Find the reactions at 'A'. 10



(b) Three cylinders are piled up in a rectangular channel as shown in **figure**. Determine the reactions between cylinders A, B and C with channel surfaces. Assume all smooth surfaces. 10

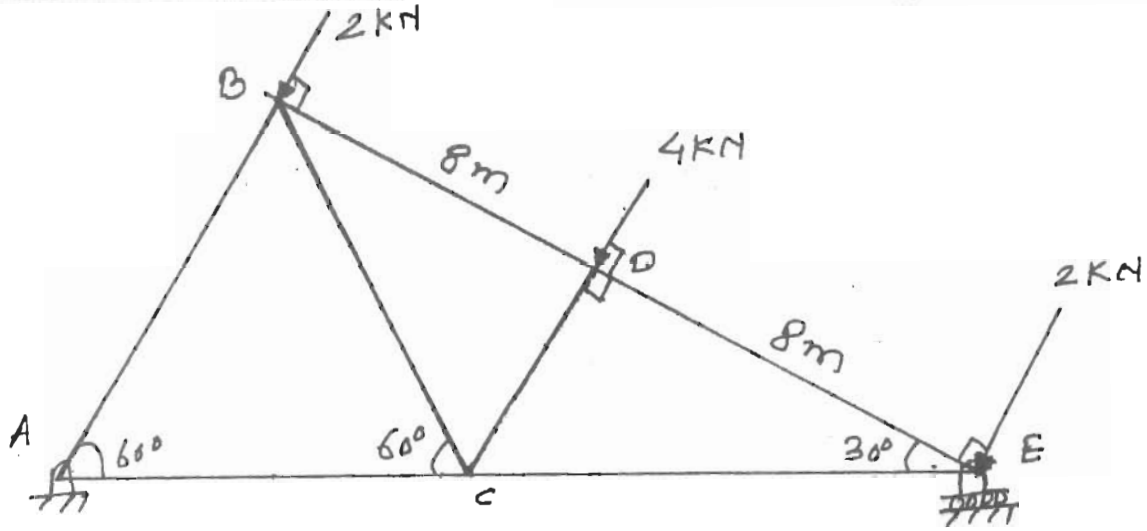


$w_A = 150\,N$, $r_A = 4\,cm$

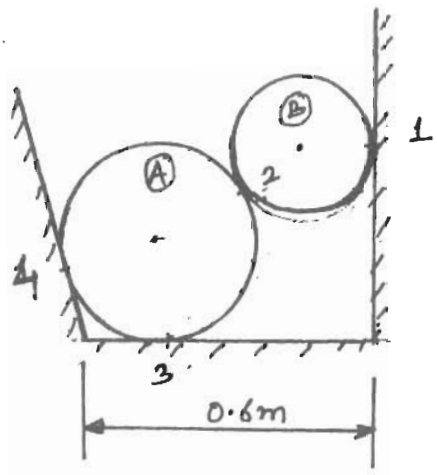
$w_B = 400\,N$ $r_B = 6\,cm$

$w_C = 200\,N$ $r_C = 5\,cm$.

3. (a) Determine forces in all the members of the plane truss as shown in **figure.** 10



(b) Two spheres rests on a smooth surfaces as shown in **figure.** Find the reactions at points of contact 1, 2, 3 and 4. 10



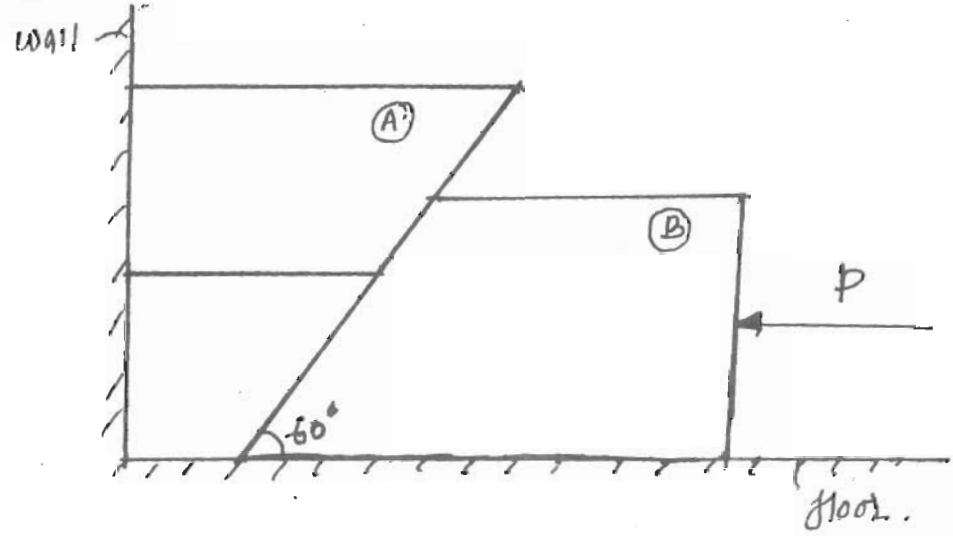
$W_A = 500\ N$

$W_B = 200\ N$

$r_A = 0.25\ m$

$r_B = 0.20\ m.$

4. (a) Two blocks A and B are resting against a wall and the floor as shown in **figure.** Find horizontal force 'P' applied to the lower block that will hold the system in equilibrium. Take $\mu = 0.25$ at floor, $\mu = 0.3$ at wall and $\mu = 0.2$ between the blocks. $W_A = 500$ N, $W_B = 1000$ N. 10



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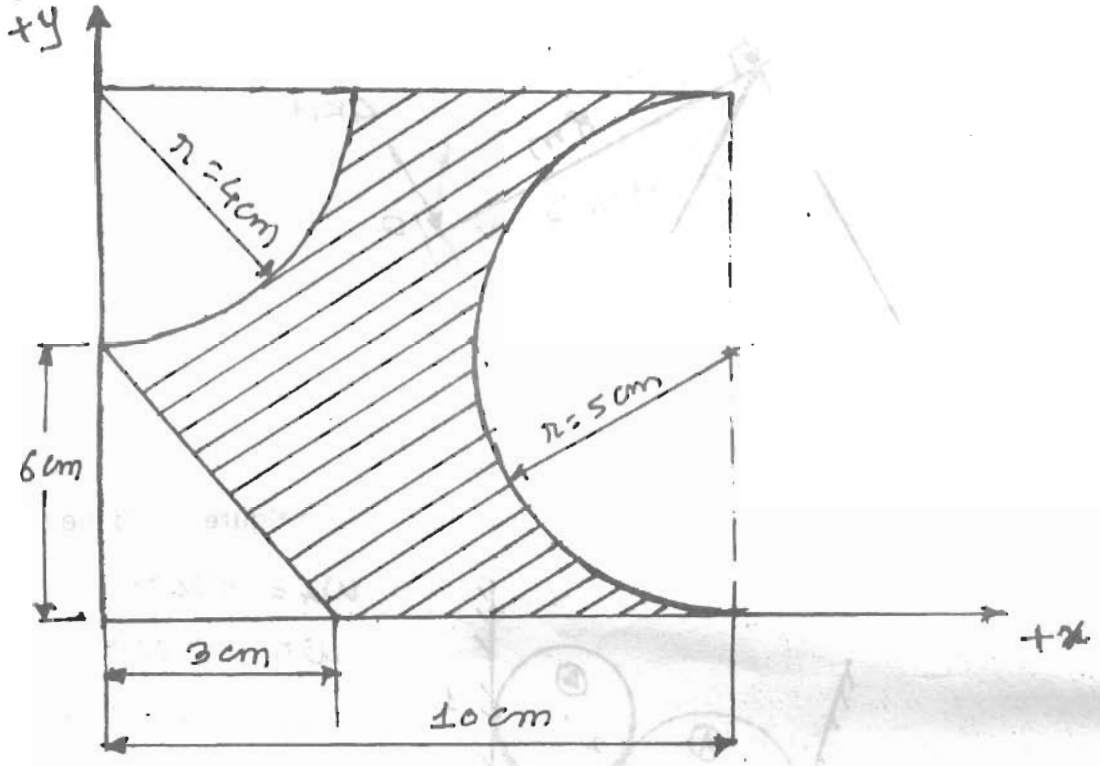
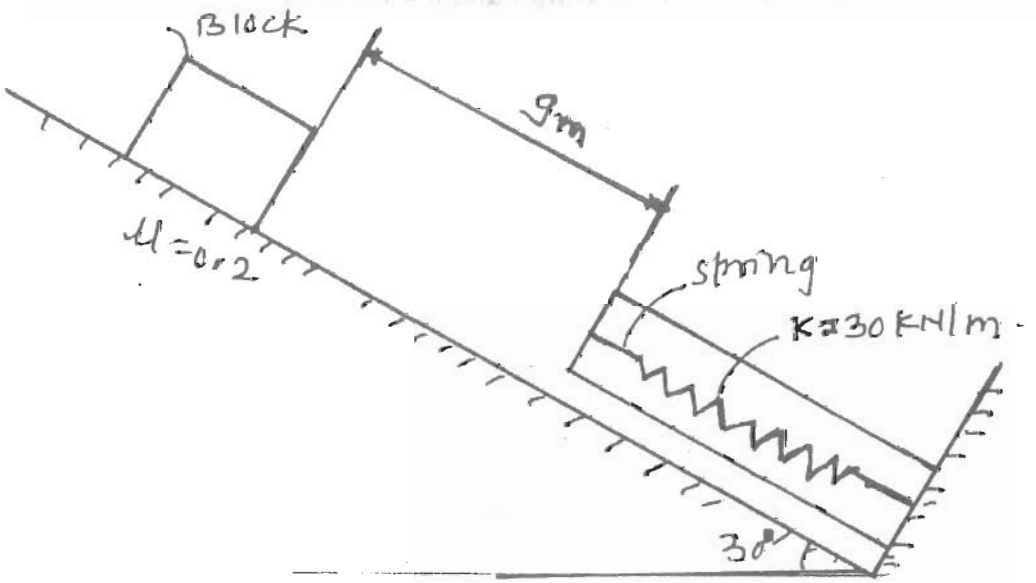
(b) Find moment of Inertia of the shaded area as shown in **figure** about its reference **x** and **y** axis. 10

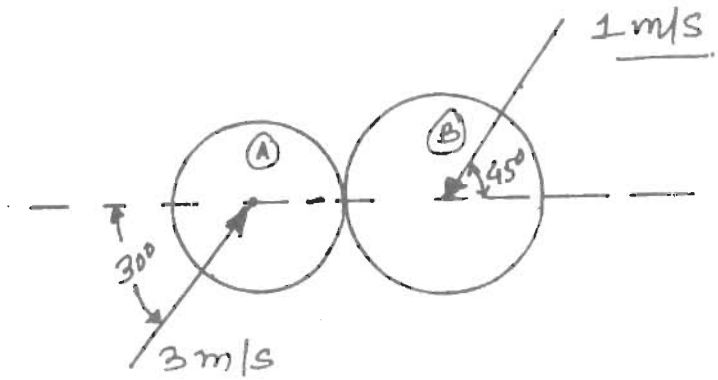


5. (a) A train leaves station **'A'** and attains speed at the rate of 4 m/s^2 for 6 sec and then 6 m/s^2 till it reaches a velocity of 48 m/s. Further the velocity remains constant, then brakes are applied giving the train a constant deacceleration stopping it in 6 sec. If the total running time between two stations is 40 sec. plot a-t, v-t and x-t curve. Determine distance between two stations. 10

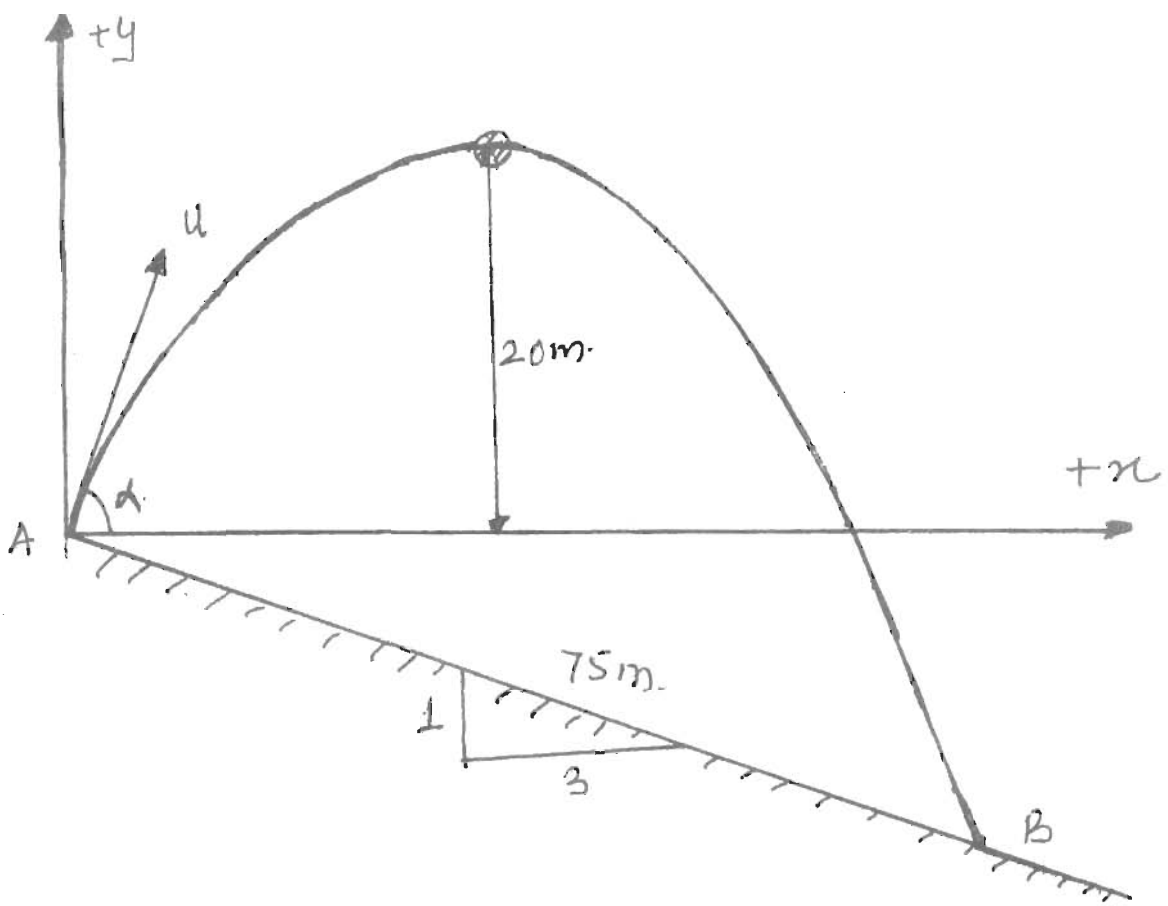
(b) A spring is used to stop 100 kg package which is moving down a 30° incline. The spring has constant k = 30 kN/m is held by cables so that it is initially compressed by 90 mm. If the velocities of package is 5 m/s, when it is 9 m from spring. Determine the maximum additional deformation of spring in bringing the package to rest. Assume coefficient of friction between block and incline as 0·2. 10



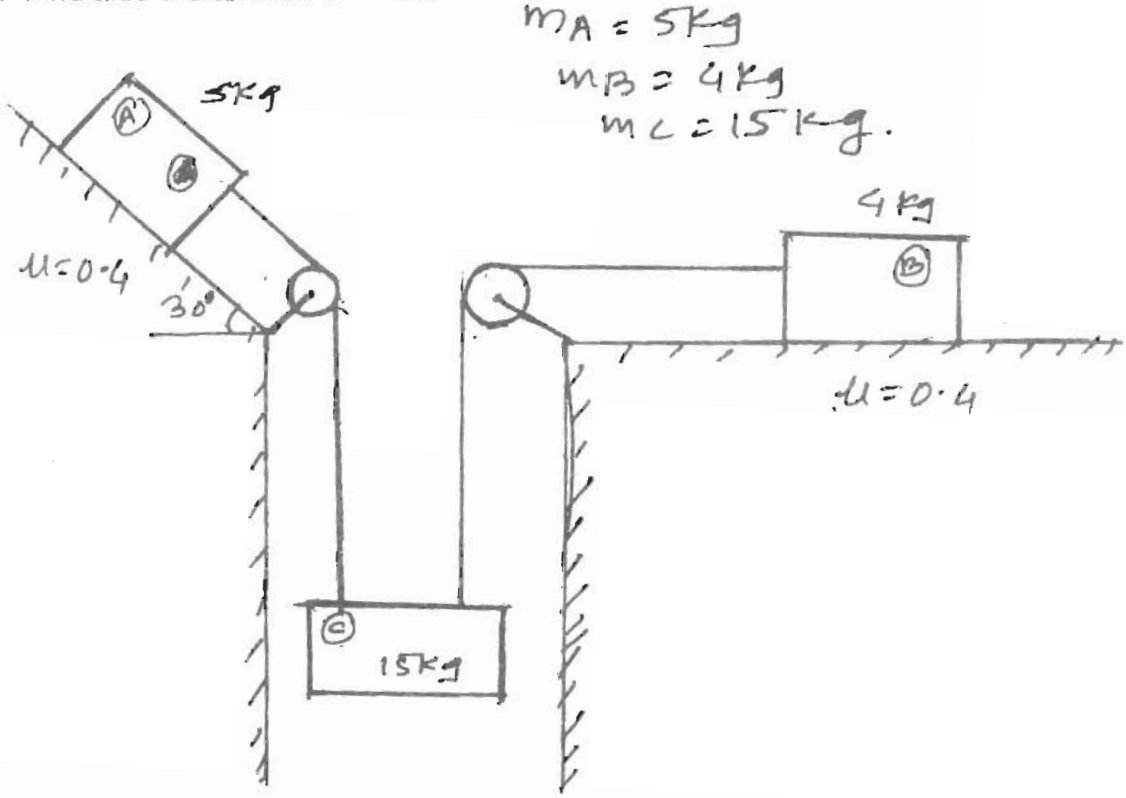
6. (a) Two smooth balls collides as shown in **figure**. If mA = 1 kg, mB = 2 kg and e = 0·75. Find the velocities after impact. **10**



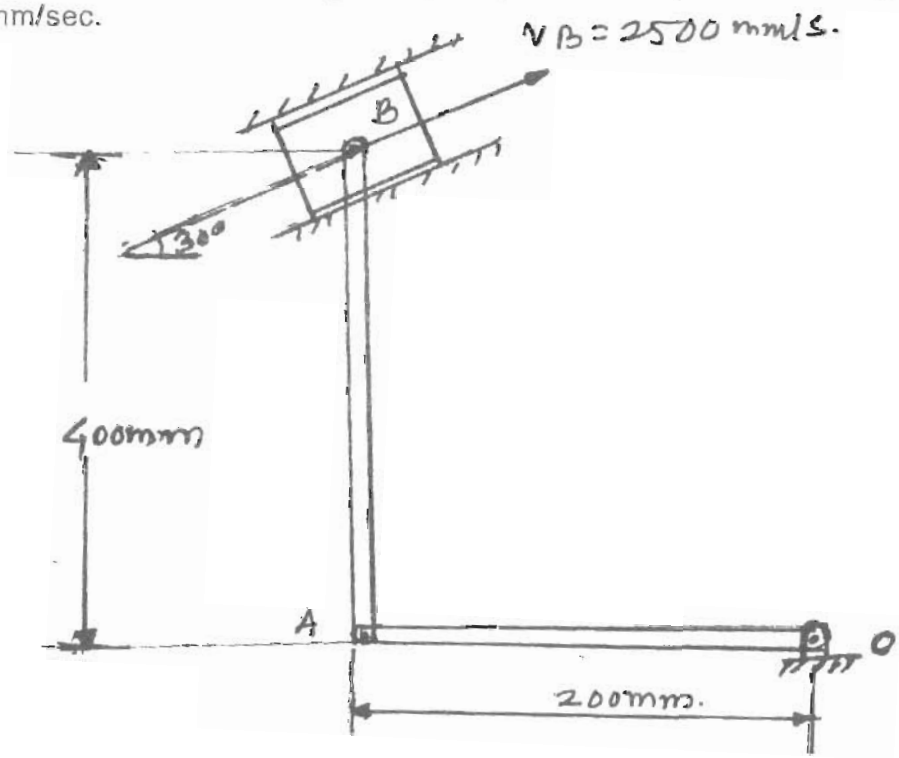
(b) A ball thrown down the incline strike the incline at a distance of 75 m along. If the ball rises 20 m above the point of projection. Complete the initial velocity and angle of projection with horizontal. **10**



7. (a) The system shown in **figure** is released from rest. What is the height lost by bodies A, B and C in 2 seconds ? Take coefficient of kinetic friction at rubbing surfaces as 0·4. Find also the tension in wires. 10



(b) For the link and slider mechanism shown in **figure**, locate the instantaneous centre of rotation of link AB. Also find the angular velocity of link 'OA'. Take velocity of slider at B = 2500 mm/sec. 10



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17/5/12 FE Sem-I (Rev.) A-Maths-I

Con. 3434-12. GN-4925

(3 Hours) [Total Marks : 100

N.B. : (1) Question **No. 1** is **compulsory.**
(2) Attempt any **four (04)** questions out of remaining **six (06)** questions.
(3) **Figures** to the **right** indicate **full** marks.

Q: 1. A) If $\text{Arg}(z+1) = \frac{\pi}{6}$ & $Arg(z-1) = \frac{2\pi}{3}$ find z. 5

B) Find n^{th} derivatie of $2^x \cdot \sin^2 x \cdot \cos^3 x$. 5

C) Expand $f(x) = x^4 - 3x^3 + 2x^2 - x + 1$ in powers of $(x-3)$. 5

D) Show that $\lim_{x \to \infty} \left(\frac{1^{1/x} + 2^{1/x} + 3^{1/x} + 4^{1/x}}{4} \right)^{4x} = 24$. 5

Q: 2. A) If $z = \log(x^2 + y^2) + \frac{x^2+y^2}{x+y} - 2\log(x+y)$, 7

prove that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \frac{x^2+y^2}{x+y}$

B) If $f(x), \emptyset(x), \varphi(x)$ are differentiable in $[a, b]$, show that 6
there exist a value c in (a, b) such that

$$\begin{vmatrix} f(a) & \emptyset(a) & \varphi(a) \\ f(b) & \emptyset(b) & \varphi(b) \\ f'(c) & \emptyset'(c) & \varphi'(c) \end{vmatrix} = 0$$

C) *If* $tan(\alpha + i\beta) = e^{i\theta}$, *prove that*
1] $\alpha = \frac{n\pi}{2} + \frac{\pi}{4}$, *and* 3
2] $\beta = \frac{1}{2} log\ tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$. 4

Q: 3. A) Show that $(1 - e^{i\theta})^{-1/2} + (1 - e^{-i\theta})^{-1/2} = \left(1 + \text{cosec}\left(\frac{\theta}{2}\right)\right)^{1/2}$. 7

B) If $f(x) = \frac{1}{x^2}$ and $g(x) = \frac{1}{x}$ then show that c of C. M. V. T. 6
is H. M. of a & b where $a > 0, b > 0$.

C) *If* $y = sin[log(x^2 + 2x + 1)]$ *then prove that* 7
$(x+1)^2 y_{n+2} + (2n+1)(x+1)y_{n+1} + (n^2+4)y_n = 0$.

Q: 4. A) Solve $x^6 - x^5 + x^4 - x^3 + x^2 - x + 1 = 0$. 7

B) If $u = lx + my$, $v = mx - ly$, prove that

1] $\left(\frac{\partial u}{\partial x}\right)_y \cdot \left(\frac{\partial x}{\partial u}\right)_v = \frac{l^2}{l^2+m^2}$ 3

2] $\left(\frac{\partial y}{\partial v}\right)_x \cdot \left(\frac{\partial v}{\partial y}\right)_u = \frac{l^2+m^2}{l^2}$. 3

C) Show that $\bar{F} = (y^2 - z^2 + 3yz - 2x)i + (3xz + 2xy)j + (3xy - 2xz + 2z)k$ 7
is both solenoidal and irrotational.

[TURN OVER

Q: 5. A) Prove that $\tan 7\theta = \frac{7\tan\theta - 35\tan^3\theta + 21\tan^5\theta - \tan^7\theta}{1 - 21\tan^2\theta + 35\tan^4\theta - 7\tan^6\theta}$ 7

B) Test the convergence of $\sum \frac{2^n+1}{3^n+n}$ 6

C) If $x = \sqrt{vw}$, $y = \sqrt{wu}$, $z = \sqrt{uv}$, and φ is a function of x, y, z 7

prove that $u\frac{\partial\varphi}{\partial u} + v\frac{\partial\varphi}{\partial v} + w\frac{\partial\varphi}{\partial w} = x\frac{\partial\varphi}{\partial x} + y\frac{\partial\varphi}{\partial y} + z\frac{\partial\varphi}{\partial z}$

Q: 6. A) Prove that $\nabla \cdot \left\{\frac{f(r)}{r}\bar{r}\right\} = \frac{1}{r^2}\frac{d}{dr}[r^2 f(r)]$. 7

hence or otherwise prove that $\text{div}(r^n\bar{r}) = (n+3)r^n$.

B) Show that $\frac{b-a}{\sqrt{1-a^2}} < \sin^{-1}b - \sin^{-1}a < \frac{b-a}{\sqrt{1-b^2}}$ hence show that

1] $\frac{\pi}{6} + \frac{\sqrt{3}}{15} < sin^{-1}\left(\frac{3}{5}\right) < \frac{\pi}{6} + \frac{1}{8}$ 3

2] $\frac{\pi}{6} + \frac{1}{2\sqrt{3}} < sin^{-1}\left(\frac{1}{4}\right) < \frac{\pi}{6} - \frac{1}{\sqrt{15}}$ 3

C) If $u = \frac{x^3y^3z^3}{x^3+y^3+z^3} + \log\left(\frac{xy+yz+zx}{x^2+y^2+z^2}\right)$ then prove that 7

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 6\frac{x^3y^3z^3}{x^3+y^3+z^3}$$

Q: 7. A) Find the extreme values of the function 7

$x^3 + 3xy^2 + 72x - 15x^2 - 15y^2$.

B) Examine the convergence of $\left(\frac{2^2}{1^2} - \frac{2}{1}\right)^{-1} + \left(\frac{3^3}{2^3} - \frac{3}{2}\right)^{-2} + \left(\frac{4^4}{3^4} - \frac{4}{3}\right)^{-3} + \cdots\cdots$ 6

C) If $\frac{(a+ib)^{x+iy}}{(a-ib)^{x-iy}} = \alpha + i\beta$ then find α & β. 7