S.E. Sem III (I.T) (old)
App. Maths-III

(OLD COURSE)

Q.P. Code : 4707

(3 Hours) **[Total Marks : 100**

N.B. : (1) Question No. 1 is compulsory.
(2) Attempt **any four** questions out of **remaining six** questions.

1 (a) Show that every square matrix A can be uniquely expressd as P+iQ, where P and Q are Hermitian matrices. **5**

(b) Evaluate $\int_0^{1+i}(x^2-iy)dz$, along the path $y=x^2$. **5**

(c) Evaluate $\int_0^{\infty}\frac{\cos 6t-\cos 4t}{t}dt$ **5**

(d) Show that sin$x$, sin3x, sin5x, ..., form a set of orthogonal elements over (o, $\pi/2$). **5**

2. (a) Reducing the following matrix to normal form and hence find its rank **6**

$$A=\begin{bmatrix}1 & 2 & -2 & 3\\ 2 & 5 & -4 & 6\\ -1 & -3 & 2 & -2\\ 2 & 4 & -1 & 6\end{bmatrix}$$

(b) Obtain the Fourler expansion of $f(x)=\left(\frac{\pi-x}{2}\right)^2$ in the interval $0\le x\le 2\pi$ and **6**

$f(x+2\pi)=f(x)$.

(c) Find the Laplace Transform of the following : **8**

(i) $\frac{\cos 2t\sin t}{e^t}$ (ii) $t\int_0^t e^{-4u}\sin 3u\,du$

2/3

3. (a) Obtain the expansion of $f(x) = x(\pi - x)$, $0 < x < \pi$ as a half range cosine series. Hence show that 6

(i) $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ (ii) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$

(b) Test for consistency and solve : 6

$$5x_1 + 3x_2 + 7x_3 = 4$$

$$3x_1 + 26x_2 + 2x_3 = 9$$

$$7x_1 + 2x_2 + 10x_3 = 5$$

(c) Find the Inverse Laplace Transform of the following : 8

(i) $\tan^{-1}\left(\frac{a}{s}\right)$ (ii) $\frac{e^{4-3s}}{(s+4)^{5/2}}$

4. (a) Find analytic function $f(z) = u+iv$, where $u+v=e^x(\cos y+\sin y)$. 6

(b) Examine for linear dependence of vectors (1, 2, -1, 0), (1, 3, 1, 2), (4, 2, 1, 0) and (6, 1, 0, 1). 6

(c) Find fourier series for $f(x) = \sqrt{1-\cos x}$, in $(0, 2\pi)$ and hence deduce that 8

$$\sum_{n=1}^{\infty} \frac{1}{4n^2 - 1} = \frac{1}{2}$$

5. (a) Using convolution theorem find the inverse Laplace Transform of 6

$$\frac{s}{(s^2 + a^2)(s^2 + b^2)}$$

(b) Evaluate $\int_c \frac{e^z}{z^2+1} dz$ over the circle $|z| = 2$. 6

(c) Find all possible Laurent's expansions of $f(z) = \frac{z^2 - 1}{z^2 + 5z + 6}$, around $z = 1$. 8

6. (a) Using Residue theorem evaluate $\int_c \frac{z^2}{(z-1)^2(z-2)}dz$, where C is $|z| = 2.5$ 6

(b) Find the Fourier expansion of 6

$$f(x)=\begin{cases}0, & -2<x<-1\\ 1+x, & -1<x<0\\ 1-x, & 0<x<1\\ 0, & 1<x<2\end{cases}$$

(c) Solve $(D^3-2D^2+5D)y=0$ with $y(0) = 0$, $y'(0) = 0$ and $y''(0) = 1$. 8

7. (a) For the following matrix A, find non-singular matrices P and Q such that PAQ is in normal form. Also find rank of A. 6

$$A=\begin{bmatrix}1 & 1 & 2\\ 1 & 2 & 3\\ 0 & -1 & -1\end{bmatrix}$$

(b) Find the Laplace Transform of $f(t) = |\sin pt|$, $t \geq 0$ 6

(c) Find the real part of an analytic function whose imaginary part is, 8

$$v = x^2 + \frac{x}{x^2+y^2} - y^2$$
