IT - Sem-III (old)
Applied Math - III

12/5/16

QP Code : **28680**

(3 Hours) [Total Marks :100

N.B. : (1) Question no. **1** is **compulsory.**
(2) Attempt any **four** questions from the remaining **six** questions.
(3) **Figures** to the **right** indicate **full** marks.
(4) **Use** of scientific calculator is **allowed.**
(5) **Answer** to **subquestions** should be **written together.**

1. (a) Find the Laplace transform of $e^{-2t}\cos 2t \sin 3t$. 5
(b) Prove that every square matrix can be uniquely expressed as a sum of symmetric and skew-symmetric matrices. 5
(c) Find the Fourier series for $f(x) = 5x$ in $(-1, 1)$. 5
(d) Evaluate $\int_0^{1+i}(x^2 - iy)dz$, along the path $y = x^2$. 5

2. (a) Reduce the following matrix to normal form and hence find its rank 6

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ -2 & 3 & 1 & 2 \\ 1 & 0 & 3 & 1 \\ 4 & 2 & 0 & 1 \end{bmatrix}$$

(b) Show that the set $S = \{\sin x, \sin 3x, \sin 5x, \ldots\}$ is orthogonal over $\left[0, \pi/2\right]$. Also find the corresponding orthonormal set. 6
(c) Find the Laplace transform of each of the following 8

(i) $t\cos^2 2t$ (ii) $\int_0^t e^{-3u}\cos 2u\,du$

3. (a) Evaluate $\int_C \frac{\sin^6 z}{\left(z - \pi/2\right)^3} dz$, where C is the circle $|z| = 2$ 6
(b) Find half range cosine series for 6

$f(x) = 1 \quad 0 < x < a/2$

$= -1 \quad a/2 < x < a$

(c) Solve $(D^2\text{-}D\text{-}2)y = 20\sin 2t$ with $y(0) = 1$ and $y'(0) = 2$ 8

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4. (a) Determine the values of a and b for which the system of equations $x+2y+3z=6$, $x+3y+5z=9$, $2x+5y+az=b$ has (i) no solution (ii) unique solution (iii) Infinite number of solutions 6

(b) Using convolution theorem find the inverse Laplace transform of 6

$$\frac{s}{\left(s^2+9\right)\left(s^2+25\right)}$$

(c) Obtain Taylor's and Laurent's expansion of $f(z)=\frac{z-1}{z^2-2z-3}$ for (i) $|z|<1$ 8

(ii) $1<|z|<3$ (iii) $|z|>3$

5. (a) Find the Laplace transform of 6

$$f(t)=a\sin pt, 0<t<\pi/p$$

$$=0 \quad , \pi/p<t<2\pi/p$$

(b) Use the adjoint method to find the inverse of $A=\begin{bmatrix}1&2&3\\2&3&0\\0&1&2\end{bmatrix}$ 6

(c) Prove that $u = x^2 - y^2 - 2xy - 2x + 3y$ is harmonic. Find a function v such that $f(z)= u + iv$ is analytic. Also express f(z) in terms of z. 8

6. (a) Expand $f(x)=\begin{cases}\pi x & 0<x<1\\ 0, & 1<x<2\end{cases}$ 6

of period 2 into a Fourier series

(b) Find two non-singular matrices P and Q such the PAQ is in the normal form 6

$$A=\begin{bmatrix}1&2&-2\\-1&3&0\\0&-2&1\end{bmatrix}$$

(c) Using Cauchy's Residue theorem, evaluate $\int_C \frac{12z-7}{(z-1)^2(2z+3)}dz$, where C is the 8

circle (i) $|z|=\frac{1}{2}$ (ii) $|z+i|=3$

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7. (a) Express the function as Heaviside's unit step function and find their Laplace transforms 6

(i) $f(t) = 0 \quad 0 < t < 1$
$= t^2 \quad 1 < t < 3$
$= 0 \quad t > 3$

(ii) $f(t) = \sin t \quad 0 < t < \pi/2$
$= \cos t \quad \pi/2 < t < \pi$
$= 0 \quad t > \pi$

(b) Prove that the matrix $A = \frac{1}{9}\begin{bmatrix} -2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & -2 & 2 \end{bmatrix}$ is orthogonal and hence find A^{-1} 6

(c) Find Fourier series for $f(x) = \sqrt{1-\cos x}, 0 < x < 2\pi$ and hence show that 8

$$\sum_{n=1}^{\infty} \frac{1}{4n^2 - 1} = \frac{1}{2}$$