S.E. Sem-III . I.T. (Old)
App. Maths - III

QP CODE : 548402

(3 Hours) [Total Marks 100]

1) Question No 1 is compulsory.
2) Attempt any four Question out of remaining questions.
3) Make suitable assumptions if required and justify the same.
4) Figures in right indicate full marks.

Q 1. a) Prove that if A is orthogonal then $|A| = \pm 1$. (5)

b) Find the Laplace transform of $L[(\sin 2t - \cos 2t)^2]$ (5)

c) Prove that $f_1(x) = 1, f_2(x) = x, f_3(x) = \frac{3x^2 - 1}{2}$ are orthogonal over $(-1,1)$. (5)

d) Construct analytic function whose real part is $e^x \cos y$. (5)

Q 2. a) Show that $\int_0^\infty \left(\frac{\sin 2t + \sin 3t}{te^t}\right) dt = \frac{3\pi}{4}$ (6)

b) Prove that there does not exists an analytic function whose real part is

$x^2 + 3x + y^2 - 4y + 6$ (6)

c) Find the Fourier series for $f(x) = 0, \quad 0 < x < \pi$

$= 2\pi - x, \quad \pi < x < 2\pi.$

Also deduce that $\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots\ldots\ldots\ldots\ldots$ (8)

Q 3. a) Find the inverse Laplace transform of $\frac{s^2}{(s^2 + a^2)^2}$ (6)

b) If $f(z) = u + iv$ is an analytic function of complex variable z and $u - v = e^x[\cos y - \sin y]$

find $f(z)$ in terms of z (6)

c) Expand f(z)= $\frac{1}{z^2(z-1)(z+2)}$ about z=0 (8)

(i) $|z| < 1$ (ii) $1 < |z| < 2$ (iii) $|z| > 2$

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Q 4. a) Evaluate $\int_c \frac{1}{z^4(z+1)}dz$ where C is $|z|=2$ (6)

b) Find Complex form of the Fourier series for $f(x)=e^{ax}$ in $-\pi<x<\pi$ where 'a' is a real constant . (6)

c) Solve $\frac{d^2y}{dt^2}+4\frac{dy}{dt}+8y=1, where\ y(0)=0, y'(0)=1$ (8)

Q 5. a) Show that the matrix A=$\frac{1}{9}\begin{bmatrix} -8 & 4 & 1 \\ 1 & 4 & -8 \\ 4 & 7 & 4 \end{bmatrix}$ is orthogonal and find its inverse. (6)

b) Solve the equations 2x+y-z=1, x-2y+3z=6, x-y+2z=9 (6)

c) Find the Laplace transform of the following

(i) $L\left[\int_0^t t\cos^2 t\,dt\right]$ (ii) $L[t\sqrt{1+\sin t}]$ (8)

Q 6. a) Using Laplace transform Show that $\int_0^\infty \left(\frac{\sin 2t+\sin 3t}{te^t}\right)dt=\frac{3\pi}{4}$ (6)

b) Find the Fourier series for $f(x)=1-x^2$, $-1\le x\le 1$ (6)

c) Find the nonsingular matrix P and Q such that PAQ is in the normal form and hence find the rank of A where (8)

A=$\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

Q 7. a) If $f(x)=c_1\phi_1(x)+c_2\phi_2(x)+c_3\phi_3(x)$ where c_1, c_2, c_3 are constants and ϕ_1, ϕ_2, ϕ_3 are orthonormal functions on (a,b) show that $\int_a^b \{f(x)\}^2\,dx=c_1^{\,2}+c_2^{\,2}+c_3^{\,2}$. (6)

b) Expand the function cosz in the Taylors series about z = $\frac{\pi}{4}$. (6)

c) Find the inverse Laplace transform of the following (8)

(i) $\frac{e^{4-3s}}{(s+4)^{\frac{5}{2}}}$ (ii) $\frac{8e^{-3s}}{(s^2+4)}$

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