SE IT/ CMPN/ CBGS

Applied Mathematics-III

**Q.P. Code :23005**

**[Time: Three Hours]**

Please check whether you have got the right question paper.

N.B:
1. Question no. 1 is compulsory.
2. Attempt any three of the remaining.
3. Figures to the right indicate full marks.

Q.1

a) Find the Laplace transform of $e^{-4t}$ sinh t sin t.

b) Find half-range sine series for f(x) = $\frac{\pi}{4}$ in $(0, \pi)$.

c) Find the values of Z for which the following function is not analytic.
Z= sin hu cos v + i cos hu sinv.

d) Show that $\nabla\left[\frac{(\bar{a}\,.\,\bar{r})}{r^n}\right] = \frac{\bar{a}}{r^n} - \frac{n(\bar{a}\,.\,\bar{r})\bar{r}}{r^{n+2}}$, where $\bar{a}$ is a constant vector.

Q.2

a) Find the inverse Z- transform of $F(z) = \frac{1}{(z-3)(z-2)}$ if $|z| < 2$.

b) Verify Laplace's equation for $u = \left(r + \frac{a^2}{r}\right)\cos\theta$ also find v and f(z).

c) Find the Fourier series for the periodic function

$$f(x) = \begin{cases} -\pi & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$

State the value of f(x) at x=0 and hence, deduce that

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$$

Q.3

a) Find $L^{-1}\left[\frac{1}{(S-3)(S-3)^2}\right]$ using convolution theorem.

b) Show that the set of functions sin x , sin 2x , sin 3x, -------------- is orthogonal
interval $[0,\pi]$

c) Verify Green's Theorem for $\int_C \bar{F}\,.\,d\bar{r}$ where $\bar{F} = x^3 i + xyj$ and c is the trian
vertices are (0,2) , (2,0) and (4,2) .

**Q.4**

a) Find Laplace transform of $f(t) = \begin{cases} a \sin pt, & 0 < t < \frac{\pi}{p} \\ 0, & \frac{\pi}{p} < t < \frac{2\pi}{p} \end{cases}$ and $f(t) = f\left(t + \frac{2\pi}{p}\right)$. 06

b) Show that $\bar{F} = (y^2 - z^2 + 3yz - 2x)i + (3xz + 2xy)j + (3xy - 2xz + 2z)k$ is both solenoidal and irrotational. 06

c) Find half range cosine series for $f(x) = x$, $0 < x < 2$. Hence deduce that $\frac{\pi^4}{90} = \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + - - - - -$ 08

**Q.5**

a) Show that $\iint_S (\nabla r^n) . d\bar{s} = n(n+1) \iiint_V r^{n-2}\, dv$ using Gauss's Divergence theorem. 06

b) Find the Z-transform of $\{k^2 e^{-ak}\}$, $k \geq 0$. 06

c) (i) Find $L^{-1}\left[\frac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)}\right]$ 08

(ii) Find $L^{-1}\left[\frac{s^2+a^2}{\sqrt{s+b}}\right]$

**Q.6**

a) Use Laplace transform to solve, $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 8y = 1$ where, $y(0) = 0$, $y'(0) = 1$ 06

b) Find the bilinear transformation which maps the points $z = \infty, i, 0$ onto the points $0, i, \infty$ respectively of w-plane. 06

c) Express the function $f(x) = \begin{cases} \frac{\pi}{2}, & for\ 0 < x < \pi \\ 0, & for\ x > \pi \end{cases}$ for Fourier Sine Integral and Show that 08

$\int_0^\infty \frac{1-\cos \pi w}{w} \ldots \frac{\pi}{}$