F.E Sem II
All branches
Applied Maths - II
08/12/07

Con. 4837-07.

CD-5482

(3 Hours) [Total Marks : 100

N.B. : (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions from remaining **six**.

1. (a) Prove that $\left|\frac{3}{2}-x\right. \left|\frac{3}{2}+x\right. = \left(\frac{1}{4}-x^2\right)\pi \sec \pi x$ provided $-1 < 2x < 1$. 5

(b) Solve $(x^2 + y^2 + 1)\, dx - 2xy\, dy = 0$. 5

(c) Show that $\int_0^\infty \frac{\log(1+ax^2)}{x}\, dx = \pi\sqrt{a}$ $(a > 0)$. 5

(d) Find the total length of the curve $x^{2/3} + y^{2/3} = a^{2/3}$. 5

2. (a) Prove that $\int_0^\pi \frac{\sin^{n-1} x}{(a+b\cos x)^n}\, dx = \frac{2^{n-1}}{(a^2-b^2)^{n/2}}\, \beta\left(\frac{n}{2}, \frac{n}{2}\right)$ 6

(b) Change the order of integration and evaluate 6

$$\int_0^1 \int_0^{\sqrt{1-y^2}} \frac{\cos^{-1}x}{\sqrt{1-x^2}\sqrt{1-x^2-y^2}}\, dx\, dy.$$

(c) Solve $(D^3 + 1)y = e^{\frac{x}{2}} \sin\left[\frac{\sqrt{3}}{2}x\right]$. 8

3. (a) Find by double integration the area enclosed by the curve $9xy = 4$ and the line $2x + y = 2$. 6

(b) Evaluate $\iint_R \frac{dx\,dy}{x^4+y^2}$ where R is the region $x \geq 1$ and $y \geq x^2$. 6

(c) Solve $(D^2 - D)y = e^x \sin x$ by method of undetermined coefficient. 8

4. (a) Find the volume bounded by $y^2 = x$, $x^2 = y$ and the planes $z = 0$ and $x + y + z = 1$. 6

(b) Evaluate $\int_0^\infty x\, e^{-x^8}\, dx \int_0^\infty x^2\, e^{-x^4}\, dx$. 6

(c) Solve $(D^2 - 1)y = 2\left(1 - e^{-2x}\right)^{\frac{-1}{2}}$ by variation of parameter. 8

5. (a) Find the mass of the lamina bounded by the curve $ay^2 = x^3$ and the line $by = x$. If the density at a point varies as the distance of the point from the x axis. 6

   (b) Verify the rule of D.U.I.S. for $\int_0^\infty e^{-at} \sin btdt$. 6

   (c) Solve $\left(\frac{d}{dx} + \frac{1}{x}\right)^2 y = \frac{1}{x^4}$. 8

6. (a) Evaluate $\iiint x^2yz \, dxdydz$ throughout the volume bounded by $x = 0$ $y = 0$ $z = 0$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. 6

   (b) Find the length of the upper arc of one loop of lemniscate $r^2 = a^2\cos2\theta$. 6

   (c) Solve $(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\cos\log(1+x)$. 8

7. (a) Change to polar co-ordinates and evaluate $\iint_R \frac{1}{\sqrt{xy}} dxdy$ where R is the region of Integration bounded by $x^2 + y^2 - x = 0$. 6

   (b) Solve $\frac{dy}{dx} = x^3 y^3 - xy$. 6

   (c) Prove that $\beta\left(n + \frac{1}{2}, n + \frac{1}{2}\right) = \frac{1}{2^{2n}} \frac{\left\lceil n + \frac{1}{2}\right.}{\left\lceil n + 1\right.} \sqrt{\pi}$ 8

   deduce that $2^n \left\lceil n + \frac{1}{2}\right. = 1.3.5....(2n-1)\sqrt{\pi}$.