F.E. Sem I (Rev.) Applied Maths-I 11/12/08.
P4/RT-Ex-08-3
All branches

Con. 3736–08. **[REVISED COURSE]** **RC–5582**

(3 Hours) **[Total Marks : 100**

N.B. : (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions out of remaining **six** questions.

1. (a) Separate into real and imaginary parts $\sqrt{i}^{\sqrt{i}}$. 5
(b) Prove that the following series is convergent – 5
$$\frac{2}{3!} + \frac{3}{4!} + \frac{4}{5!} + \ldots\ldots$$
(c) If f(x) and g(x) are respectively e^x and e^{-x}, prove that c of Cauchy's mean value theorem is the arithmatic mean of a and b. 5
(d) Solve $x^4 + i = 0$. 5

2. (a) Find the unit normal to the surface $xy^3z^2 = 4$ at (–1, –1, 2). 6
(b) If $y = x^n \log x$, prove that $y_{n+1} = \frac{n!}{x}$. 6
(c) Solve (i) 7 cosh x + 8 sinh x = 1 for real values of x. 4
(ii) $\tanh x = \frac{1}{2}$. 4

3. (a) Examine the function $f(x, y) = y^2 + 4xy + 3x^2 + x^3$ for extreme values. 6
(b) Prove that $\sec^2 x = 1 + x^2 + \frac{2x^4}{3} + \ldots..$ 6
(c) Show that for real values of a and b, 8
$$e^{2ai\cot^{-1}b}\left[\frac{bi-1}{bi+1}\right]^{-a} = 1$$

4. (a) If $f(x, y) = (50 - x^2 - y^2)^{1/2}$, find the approximate value of [f(3,4) – f(3·1, 3·9)] 6
(b) If $x = \cos\theta + i\sin\theta$, $y = \cos\phi + i\sin\phi$ prove that – 6
$$\frac{x-y}{x+y} = i\tan\left(\frac{\theta-\phi}{2}\right)$$
(c) Prove that $\nabla\cdot\left\{\frac{f(r)}{r}\bar{r}\right\} = \frac{1}{r^2}\frac{d}{dr}\left[r^2 f(r)\right]$ 8

Hence, or otherwise prove that $\mathrm{div}\left(r^n\ \bar{r}\right) = (n+3)r^n$.

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5. (a) Evaluate $\lim_{x\to 0} \log_{\tan x} \tan 2x$. 6

(b) If $\alpha - i\beta = \dfrac{1}{a - ib}$, prove that $(\alpha^2 + \beta^2)(a^2 + b^2) = 1$ 6

(c) Verify Euler's Theorem for $u = x^2 \tan^{-1}\left(\dfrac{y}{x}\right) - y^2 \tan^{-1}\left(\dfrac{x}{y}\right)$ and also prove that 8

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{x^2 - y^2}{x^2 + y^2}$$

6. (a) If $u = f(e^{y-z}, e^{z-x}, e^{x-y})$, then prove that 6

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$$

(b) If $y = \cos(m \sin^{-1}x)$, 6

Prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)\, y_n = 0$.

(c) If w is a 7th root of unity prove that 8

$s = 1 + w^n + w^{2n} + w^{3n} + w^{4n} + w^{5n} + w^{6n} = 7$ if n is a multiple of 7 and is equal to zero otherwise.

7. (a) Prove that $\log(1 + x) = \dfrac{x}{1+\theta x}$, where $0 < \theta < 1$ and hence deduce that 6

$$\frac{x}{1+x} < \log(1+x) < x, \quad x > 0.$$

(b) Prove that if the sum and product of two complex numbers are real then the two numbers must be either real or conjugate. 6

(c) Prove that $e^{\cos^{-1}x} = e^{\pi/2}\left[1 - x + \dfrac{x^2}{2} - \dfrac{x^3}{3} + \ldots\right]$ 8