F.E. Sem II (Rev.) All branches   Applied Maths II   08/12/08

Con. 3742–08.   **(REVISED COURSE)**   **RC–5562**

(3 Hours)   **[Total Marks : 100**

N.B. : (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions out of remaining **six** questions.
(3) **Figures** to the **right** indicates **full** marks.

1. (a) Show that $\int_0^1 \sqrt{1-x^4}\, dx = \frac{\sqrt{\pi}}{6} \frac{\Gamma\left(\frac{1}{4}\right)}{\Gamma\left(\frac{3}{4}\right)}$ 5

(b) Find the total length of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ 5

(c) Change the order of integration and evaluate $\int_0^5 \int_{2-x}^{2+x} dx\, dy$. 5

(d) Solve $6\frac{d^2y}{dx^2} + 17\frac{dy}{dx} + 12y = e^{\frac{-3x}{2}} + 2^x$ 5

2. (a) Evaluate $\int_0^{\pi} \frac{dx}{a+b\cos x}$, a > 0, b > 0 and deduce 8

that $\int_0^{\pi} \frac{dx}{(a+b\cos x)^2} = \frac{\pi a}{\left(a^2-b^2\right)^{3/2}}$

(b) Show that $\int_0^a \frac{dx}{\left(a^n - x^n\right)^{1/n}} = \frac{\pi}{n} \operatorname{cosec}\left(\frac{\pi}{n}\right)$ 6

(c) Evaluate $\iiint x^2yz\; dx\, dy\, dz$ throughout the valume bounded by the planes 6

$x = 0,\ y = 0,\ z = 0,\ \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1.$

3. (a) Solve $\left(\frac{y}{x}\sec y - \tan y\right)dx - (x - \sec y \log x)\,dy = 0$ 6

(b) Using Eulers Method find approximate value of y at x = 1 in five steps taking 6

$h = 0{\cdot}2$, Given $\frac{dy}{dx} = x + y$ and y(0) = 1

(c) Solve $(D^3 + D)\, y = \operatorname{cosec} x$, by method of variation of parameters. 8

4. (a) Solve by Runge–Kutta Method of fourth order $\frac{dy}{dx} = 3x + y^2$, $x_0 = 1$, $y_0 = 1{\cdot}2$ at $x = 1{\cdot}1$. 8

(b) Solve by Taylors Series Method $\frac{dy}{dx} = -xy$ with $x_0 = 0$, $y_0 = 1$. 6

(c) Solve $(x+2)^2 \frac{d^2v}{dx^2} - (x+2)\frac{dv}{dx} + v = 3x+4$. 6

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5. (a) Find the area of the Cardioide, $r = a(1 + \cos\theta)$. 6

(b) Solve $\frac{dy}{dx}\cosh x = 2\cosh^2 x \sinh x - y \sinh x$. 6

(c) (i) Solve $\frac{dy}{dx} = 1 - x(y - x) - x^3(y - x)^2$ 4

(ii) Solve $\frac{d^4y}{dx^4} - a^4y = \sin ax$. 4

6. (a) Evaluate $\iint_R xy\,(x+y)\,dxdy$ where R is the region bounded by $xy = 4$, $y = 0$, $x = 1$, $x = 4$. 6

(b) Evaluate $\int_0^{\pi/4}\int_0^{\sqrt{\cos 2\theta}} \frac{r}{(1+r^2)^2}\,d\theta dr$ 6

(c) Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$. 8

7. (a) Evaluate $\int_{-\pi}^{\pi} \sin^2 x \cos^4 x\,dx$ 6

(b) (i) The differential equation of motion of a body is $\frac{d^2x}{dt^2} + n^2x = t\cos it$ 4

Solve this equation, what is the solution if $i = n^2$.

(ii) The density of a uniform circular lemina of radius 'r' varies as the square of its distance from a fixed point on the circumference of the circle. Find the mass of the lemina. 4

(c) Sketch the area of integration and evaluate 6

$$\int_1^2 \int_{-\sqrt{2-y}}^{\sqrt{2-y}} 2x^2y^2\,dxdy$$