JT | CMPN App. Maths II (012) 18/12/12

ws-Sept, 2012 (c) 153

Con. 11025–12. **KR–8886**

(3 Hours) **[Total Marks : 100**

N. B. : (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions from Question Nos. **2** to **7**.

1. (a) A random variable X has the distribution – **5**

X	0	1	2	3	4	5	6
P(x)	K	3K	5K	7K	9K	11K	13K

Find : (i) K (ii) $P\{X < 4\}$ (iii) $P\{3 < X \le 6\}$

(b) A box contains 100 transistors, out of which 20 are defectives. If 10 are selected for inspection, find the probability that – (i) at least one is defective (ii) at most 3 are defective. **5**

(c) Solve the given LPP using graphical method. **5**

Minimize $Z = 20x_1 + 30x_2$

Subject to
$x_1 + 2x_2 \le 40$
$3x_1 + x_2 \ge 30$
$4x_1 + 3x_2 \ge 60$
$x_1, x_2 \ge 0$

(d) Write the dual of the given LPP – **5**

Minimize $Z = x_1 - x_2 + 3x_3$

Subject to
$x_1 + x_2 + x_3 \le 10$
$2x_1 - x_3 \ge 2$
$2x_1 - 2x_2 + 3x_3 = 6$
$x_1, x_3 \ge 0$, x_2 unrestricted.

2. (a) If $f(x) = \begin{cases} 2e^{-2x} & x \ge 0 \\ 0 & \text{otherwise} \end{cases}$ is a pdf of random variable. Find first four moments about origin. **6**

(b) Samples of two types of electric bulbs were tested for length of life and following data were obtained. **6**

Type I : $n_1 = 8$, $\bar{x}_1 = 1234$ hrs $s_1 = 36$ hrs

Type II : $n_2 = 7$, $\bar{x}_2 = 1036$ hrs, $s_2 = 40$ hrs

Is type I better than type II ? Test at 5% significance level.

(c) Find the equation of lines of regression of (i) y on x and (ii) x on y. **8**

x	5	6	7	8	9	10	11
y	11	14	14	15	12	17	16

[TURN OVER

3. (a) Suppose X follows Poisson distribution with parameter λ and 6

$P\{X = 2\} = \frac{2}{3} P\{X = 1\}$.

Find $P\{X = 3\}$.

(b) Find all basic solution to the LPP and classify them as feasible, degenerate and optimal. 6

Maximize $Z = 2x_1 + 4x_2 + x_3 + x_4$

Subject to $x_1 + 3x_2 + x_4 = 4$

$2x_1 + x_2 = 3$

$x_2 + 4x_3 + x_4 = 5$

$x_1, x_2, x_3, x_4 \geq 0$

(c) Use the Kuhn-Tucker conditions to solve the non-linear problem — 8

Minimize $Z = 2x_1 + 3x_2 - x_1^2 - 2x_2^2$

Subject to $2x_1 + 3x_2 \leq 6$

$5x_1 + 2x_2 \leq 10$

$x_1, x_2 \geq 0$

4. (a) Obtain the rank corelation coefficient of the given data : 6

x	10	12	18	18	15	40	18
y	12	18	25	25	30	25	15

(b) The marks of 1000 students of an Engineering college are distributed normally with mean 70 and standard deviations 5. Estimate the number of students whose marks will be (i) between 60 and 75 (ii) more than 75. 6

(c) Four coins are tossed and the number of heads is noted. The experiment is repeated 100 times and the following distribution is obtained : 8

No. of heads	0	1	2	3	4
Frequency	7	18	40	31	4

Does this result support that all four coins are unbiased ? Test at 5% level of significance.

5. (a) An insurance agent has claimed that the average age of policy holders who insure through him is less than the average age for all agents which is 30·5 years. A random sample of 100 policy holders who had insured through him gave the distribution. 6

Age of insurance	21–25	26–30	31–35	36–40
No. of insurance	32	22	30	16

Test his claim at 2% significance level.

(b) Use the Simplex method to solve the LPP 6

$$\begin{aligned}\text{Maximize } Z &= 2x_1 + 4x_2 + x_3 + x_4\\ \text{subject to } & x_1 + 3x_2 + x_4 \leq 4\\ & 2x_1 + x_2 \leq 3\\ & x_2 + 4x_3 + x_4 \leq 3\\ & x_1, x_2, x_3 \geq 0\end{aligned}$$

(c) Using the method of Lagrangian multipliers, solve the NLPP and apply bordered Hessian matrix to test whether stationary point maximizes or minimises the objective function. 8

$$\begin{aligned}\text{optimize } Z &= 2x_1^2 + 3x_2^2 + x_3^2\\ \text{subject to } & x_1 + x_2 + 2x_3 = 13\\ & 2x_1 + x_2 + x_3 = 10\\ & x_1, x_2, x_3 \geq 0\end{aligned}$$

6. (a) The manager of a chain of restaurants take a random sample of 100 customers. He categorizes them according to their service satisfaction and salary of the waiter who had served them. 6

	Waiter Low	**Salary High**
Service Good	28	24
Quality Poor	19	29

Test whether the quality of service is independent of the waiter's salary. Apply 5% L.S.

(b) Use Big M penalty method to solve the LPP 6

$$\begin{aligned}\text{Maximize } Z &= 6x_1 + 4x_2\\ \text{subject to } & 2x_1 + 3x_2 \leq 30\\ & 3x_1 + 2x_2 \leq 24\\ & x_1 + x_2 \geq 3\\ & x_1, x_2 \geq 0\end{aligned}$$

(c) Fit a least square parabola of y on x. 8

x	1·2	1·8	3·1	4·9	5·7	7·7	8·6	9·8
y	4·5	5·9	7·0	7·8	7·2	6·8	4·5	2·7

7. (a) Use dual simplex method to solve the LPP 6

$$\text{Maximize } Z = -2x_1 - 2x_2 - 4x_3$$
$$\text{subject to } 2x_1 + 3x_2 + 5x_3 \geq 2$$
$$3x_1 + x_2 + 7x_3 \leq 3$$
$$x_1 + 4x_2 + 6x_3 \leq 5; \quad x_1, x_2, x_3 \geq 0$$

(b) Find the expectation of numbers of tosses required to obtain a six first time in a series of independent tosses of a die. 6

(c) Ten school boys were given a test in mathematics. They were given a months special coaching and a second test was given to them in the same subject. 8

Marks in Test I	70	68	55	75	80	90	68	75	56	58
Marks in Test II	68	70	52	74	75	78	80	92	54	55

Test if the marks given above give evidence that the student are benefitted by coaching. Use 5% level of significance.