IT Sem III (OLD) AM-III 20/11/15

Q.P. Code : **1084**

(3 Hours) [Total Marks : 100

N.B. : (1) Question No. 1 is **compulsory**.
(2) Attempt any **four** questions from the remaining six questions.
(3) **Figures** to the **right** indicate **full** marks.

1 (a) If $f(z) = u + iv$ is analytic & $u - v = e^x(\cos y - \sin y)$, find $f(z)$ in terms of z. 5
(b) Find half range sine series in $(0, \pi)$ for $x(\pi - x)$ 5
(c) Find Laplace Transformation of cost.cos2t.cos3t. 5
(d) Is the following matrix orthogongal matrix ? If yes, how ? 5

$$A = \begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$$

2. (a) Investigate for what value of λ and μ the equations $2x+3y+5z=9$; $7x+3y-2z=8$; $2x+3y+\lambda z=\mu$ have (i) no solution (ii) a unique solution (iii) an infinite no of solutions. 6

(b) Verify Laplaces equation for $u = \left(r + \frac{a^2}{r}\right)\cos\theta$ Also find v & f(z) 6

(c) Use Laplace transform to solve, $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 8y = 1$, where $y(0) = 0$, $y'(0) = 1$. 8

3. (a) Evaluate $\oint_C \frac{z+3}{2z^2+3z-2}dz$ where C is the circle $|z - i| = 2$ 6

(b) Using convolution theorem prove that, $L^{-1}\left\{\frac{1}{s}\log\left(\frac{s+1}{s+2}\right)\right\} = \int_C^1 \frac{e^{-2u} - e^{-u}}{u} du$ 6

(c) Obtain Fourier series of $x \cos x$ in $(-\pi, \pi)$ 8

4. (a) Show that $u = \cos x \cosh y$ is harmonic function. Find its harmonic conjugate & corresponding analytic function 6

(b) Show that the set of functions $\frac{\cos x}{\sqrt{\pi}}, \frac{\cos 2x}{\sqrt{\pi}}, \frac{\cos 3x}{\sqrt{\pi}}, \ldots\ldots$ form a orthnormal set in the interval $(-\pi, \pi)$ 6

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(c) Find non singular matrices P & Q such that PAQ is in normal form. Hence find the rank of matrix A, where $A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$ 8

5. (a) Using Residue theorem, evaluate, $\int_0^{2\pi} \frac{d\theta}{5 + 3\sin\theta}$ 6

(b) Every square matrix A can be uniquely expressed as P+ iQ, where P & Q are Hermitian matrices. 6

(c) Find half range cosine series for $f(x) = x$; $0 < x < 2$. Using parseval's identity deduce that (i) $\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots\ldots$ 8

(ii) $\frac{\pi^4}{90} = \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} \ldots\ldots$

6. (a) Evaluate $\int_0^{\infty} \frac{\cos at - \cos bt}{t} dt$ 6

(b) Expand $f(x) = \pi x \quad 0 < x < 1$
$\quad = 0 \quad 1 < x < 2$
with period 2, into a Fourier series. 6

(c) Obtain two distinct Laurent's series for $f(z) = \frac{2z-3}{z^2 - 4z + 3}$ in powers of (z - 4) 8

7. (a) Find the matrix A, if adj $A = \begin{bmatrix} -2 & 1 & 3 \\ -2 & -3 & 11 \\ 2 & 1 & -5 \end{bmatrix}$ 6

(b) Find fourier series of $f(x) = 1-x^2$ in (-1 , 1) 6

(c) (i) Find the Laplace Transform of $(e^{-4t}\sinh t \sin t)$ 8

(ii) Find $L\left\{\int_0^t u\cos^2 u\, du\right\}$
