

(3 Hours)

[Total Marks : 80

- N.B. :** (1) Question No. **one** is **compulsory**.
 (2) Answer **any three** questions from **Q.2 to Q.6**
 (3) Use of statistical Tables permitted.
 (4) Figures to the **right** indicate **full** marks
 (5) Assume suitable data wherever applicable.

1. (a) Find the Eigenvalues and eigenvectors of the matrix.

5

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

- (b) Evaluate the line integral $\int_0^{1+i} (x^2 + iy) dz$ along the path $y = x$

5

- (c) Find k and then $E(x)$ for the p.d.f.

5

$$f(x) = \begin{cases} k(x-x^2), & 0 \leq x \leq 1, k > 0 \\ 0, & \text{otherwise} \end{cases}$$

- (d) Calculate Karl Pearson's coefficient of correlation from the following data.

5

x	100	200	300	400	500
y	30	40	50	60	70

2. (a) Show that the matrix $A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ is non-derogatory.

6

- (b) Evaluate $\int \frac{e^{2z}}{(z+1)^4} dz$ where C is the circle $|z-1|=3$

6

- (c) If x is a normal variate with mean 10 and standard deviation 4 find
 (i) $P(|x-14| < 1)$ (ii) $P(5 \leq x \leq 18)$ (ii) $P(x \leq 12)$

8

3. (a) Find the relative maximum or minimum (if any) of the function 6
 $Z = x_1^2 + x_2^2 + x_3^2 - 4x_1 - 8x_2 - 12x_3 + 100$
- (b) If x is Binomial distributed with $E(x) = 2$ and $V(x) = 4/3$, find the probability distribution of x . 6
- (c) If $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, find A^{50} . 8
4. (a) Solve the following L.P.P. by simplex method 6
 Minimize $z = 3x_1 + 2x_2$
 Subject to $3x_1 + 2x_2 \leq 18$
 $0 \leq x_1 \leq 4$
 $0 \leq x_2 \leq 6$
 $x_1, x_2 \geq 0$.
- (b) The average of marks scored by 32 boys is 72 with standard deviation 8 while that of 36 girls is 70 with standard deviation 6. Test at 1% level of significance whether the boys perform better than the girls. 6
- (c) Find Laurent's series which represents the function $f(z) = \frac{2}{(z-1)(z-2)}$ 8
 When (i) $|z| < 1$, (ii) $1 < |z| < 2$ (iii) $|z| > 2$
5. (a) Evaluate $\int_C \frac{z^2}{(z-1)^2(z+1)} dz$ where C is $|z| = 2$ using residue theorem 6
- (b) The regression lines of a sample are $x+6y=6$ and $3x+2y=10$ Find 6
 (i) Sample means \bar{x} and \bar{y}
 (ii) Correlation coefficient between x and y . Also estimate y When $x = 12$
- (c) A die was thrown 132 times and the following frequencies were observed 8

No. obtained	1	2	3	4	5	6	Total
Frequency	15	20	25	15	29	28	132

Using χ^2 -test examine the hypothesis that the die is unbiased.

6. (a) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 + x + 2}{x^4 + 10x^2 + 9} dx$ using contour integration. 6

(b) If a random variable x follows Poisson distribution such that $P(x=1) = 2P(x=2)$ Find the mean and the variance of the distribution. Also find $P(x=3)$. 6

(c) Use Penalty method to solve the following L.P.P. 8

Minimize $z = 2x_1 + 3x_2$
Subject to $x_1 + x_2 \geq 5$
 $x_1 + 2x_2 \geq 6$
 $x_1, x_2 \geq 0.$

- N.B. :** (1) Question No. 1 is **compulsory**
 (2) Attempt any **three** questions from remaining questions
 (3) **Draw suitable diagrams** wherever **necessary**
 (4) **Assume suitable data**, if **necessary**.

1. (a) Design a DFA over an alphabet $\Sigma = \{a, b\}$ to recognize a language in which every 'a' is followed by 'b'. **5**
 (b) Give formal definition of a Push Down Automata. **5**
 (c) State and explain the power and limitations of a Turing machine **5**
 (d) Design a mealy machine to determine the residue mod 3 of a binary number. **5**

2. (a) Convert the following NFA to an equivalent DFA **10**

State	a	b	ϵ
$\rightarrow q_0$	$\{q_0, q_1\}$	q_1	$\{\}$
q_1	$\{q_2\}$	$\{q_1, q_2\}$	$\{\}$
$*q_2$	$\{q_0\}$	$\{q_2\}$	$\{q_1\}$

- (b) State and explain pumping lemma for regular languages. Using pumping lemma **10**
 prove that the language $L = \{0^n 1^n \mid n \geq 0\}$ is not regular.

3. (a) Design a Turing machine that computes a function $f(m,n) = m + n$ i.e. addition of two integers **10**
 (b) Design a Turing machine to accept the language $0^n 1^n 2^n$ **10**

4. (a) Draw a state diagram and construct a regular expression corresponding to the following state transition table. **10**

State	0	1
$\rightarrow *q_1$	q_1	q_2
q_2	q_3	q_2
q_3	q_1	q_2

- (b) State and explain decision properties of regular languages **10**

[Turn Over