Con. 2941-08. **CO-5104**

(REVISED COURSE)

(3 Hours) **[Total Marks : 100**

N.B. : (1) Question No. **1** is **Compulsory**.
(2) Attempt any **four** questions out of remaining **six** questions.

1. (a) If $u = \log\left(\tan\left[\frac{\pi}{4} + \frac{\theta}{2}\right]\right)$ then P.T. **20**

   (i) $\cosh u = \operatorname{Sec} \theta$

   (ii) $\sinh u = \tan \theta$.

   (b) Find the complex number 'z' if—

   $$\arg (z + 1) = \frac{\pi}{6} \text{ and } \arg (z - 1)\ \frac{2\pi}{3}.$$

   (c) If $u = (1 - 2xy + y^2)^{-1/2}$, P.T. $x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = y^2u^3$.

   (d) If $\bar{u} = a \cos t\ i + a \sin t\ j + at \tan\alpha\ k$

   then S.T. $\left[\frac{d\bar{u}}{dt}\ \ \frac{d^2\bar{u}}{dt^2}\ \ \frac{d^3\bar{u}}{dt^3}\right] = a^3 \tan\alpha.$

2. (a) If $y = e^{m\sin^{-1}x}$ then P.T. **6**

   $$(1 - x^2)\, y_{n+2} - (2n + 1)\, xy_{n+1} - (n^2 + m^2)\, y_n = 0.$$

   (b) Find the maximum and minimum values of— **6**

   $x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$.

   (c) If $z = f(x, y)$, $x = e^u \cos v$, $y = e^u \sin v$ **8**

   P.T. (i) $x\frac{\partial z}{\partial v} + y\frac{\partial z}{\partial u} = e^{2u}\frac{\partial z}{\partial y}$

   (ii) $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = e^{-2u}\left[\left(\frac{\partial z}{\partial u}\right)^2 + \left(\frac{\partial z}{\partial v}\right)^2\right]$

3. (a) Prove that $\nabla(f(r)) = f'(r)\frac{\bar{r}}{r}$ and hence find f if $\nabla f = 2r^4\bar{r}$. **6**

   (b) Find the values of a, b, c so that— **6**

   $$\lim_{x \to 0} \frac{ae^x - b\cos x + ce^{-x}}{x \sin x} = 2.$$

   (c) If $\cos\alpha + \cos\beta + \cos y = 0$ and $\sin\alpha + \sin\beta + \sin y = 0$ then P.T. :— **8**

   (i) $\sin^2\alpha + \sin^2\beta + \sin^2 y = \cos^2\alpha + \cos^2\beta + \cos^2 y = \frac{3}{2}$.

   (ii) $\cos(2\alpha) + \cos(2\beta) + \cos(2y) = 0$

   (iii) $\cos(\alpha + \beta) + \cos(\beta + y) + \cos(y + \alpha) = 0$

   (iv) $\sin(\alpha + \beta) \sin(\beta + y) + \sin(y + \alpha) = 0$.

4. (a) If $z = f(x, y)$, $x = r\cos\theta$, $y = r\sin\theta$, P.T. **6**

$$\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2=\left(\frac{\partial z}{\partial r}\right)^2+\frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$$

(b) If $x + iy = c\cot(u + iv)$, Show that— **6**

$$\frac{x}{\sin(2u)}=\frac{-y}{\sinh(2v)}=\frac{c}{\cosh(2v)-\cos(2u)}$$

(c) If $\bar{a}$ is a constant vector and $\bar{r} = xi + yj + zk$ **8**

P.T. (i) $\operatorname{div}(\bar{a}\times\bar{r})=0$

(ii) $\operatorname{div}(\bar{a}\cdot\bar{r})\bar{a}=a^2$

(iii) $(\bar{a}\times\bar{r}\times\bar{a})=2a^2$

(iv) $\operatorname{curl}(\bar{a}\times\bar{r})=2\bar{a}$.

5. (a) P.T. $\tan^{y} x = x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+.......$ **6**

(b) P.T. $\frac{b-a}{b}<\log\left(\frac{b}{a}\right)<\frac{b-a}{a}$ for $0<a<b$ **6**

Hence deduce that $\frac{1}{4}<\log\frac{4}{3}<\frac{1}{3}$.

(c) Separate into real and imaginary parts $\tan^{-1}(\cos\theta+\sin\theta)$ **8**

6. (a) Test the convergence of— **6**

$$\frac{x}{1\cdot2}+\frac{x^2}{3\cdot4}+\frac{x^3}{5\cdot6}+\frac{x^4}{7\cdot8}+......\quad(x>0\text{ and }x\neq1)$$

(b) If $u = f(y/x)+\sqrt{x^2y^2}$ **6**

then P.T. $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sqrt{x^2+y^2}$.

(c) If $y = 2^x\cos^9 x$ then find y_n. **8**

7. (a) Find all roots of $(x + 1)^7 = (x - 1)^7$. **6**

(b) If $u=\operatorname{cosec}^{-1}\left(\sqrt{\frac{x^{1/2}+y^{1/2}}{x^{1/3}+y^{1/3}}}\right)$ then P.T. **6**

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\,\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{\tan u}{12}\left[\frac{13}{12}+\frac{\tan^2 u}{12}\right]$$

(c) If $z = x\log(x + r) - r$ where $r^2 = x^2 + y^2$ **8**

P.T. (i) $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=\frac{1}{x+r}$

(ii) $\frac{\partial^2 z}{\partial x^3}=-\left(\frac{x}{r^3}\right)$