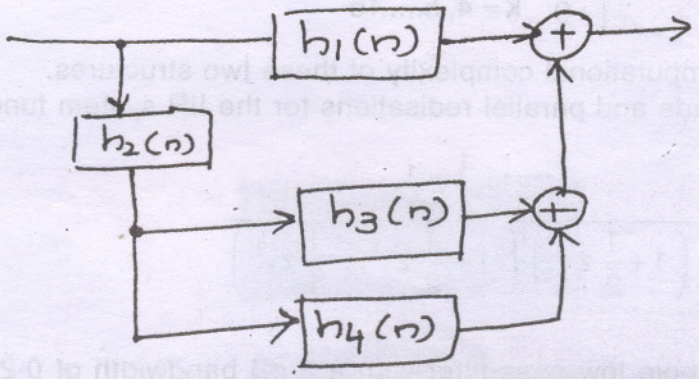
N.B. (1) Question No. **1** is **compulsory.**
(2) Attempt any **four** questions from question Nos. **2** to **7.**
(3) Assume **suitable** data if **required,** stating them **clearly.**

1. (a) Consider the discrete-time system given in **figure**. Composed of an interconnection of four simple discrete-time systems. **20**



$h_1(n) = \delta(n) + \frac{1}{2}\,\delta(n-1)$ $\qquad$ $h_2(n) = \frac{1}{2}\,\delta(n) + \frac{1}{4}\,\delta(n-1)$

$h_3(n) = 2\,\delta(n)$ $\qquad$ $h_4(n) = -2\left(\frac{1}{2}\right)^n u(n)$

Find the overall impulse response h(n).

(b) Determine the convolution of the two sequences :

$x(n) = \{ 3, -2, 4 \}$ and $h(n) = \{ 4, 2, -1 \}$
$\qquad\qquad\quad\uparrow \qquad\qquad\qquad\uparrow$

(c) Prove that for two finite-energy sequences x(n) and y(n), their cross-correlation sequence : Satisfy the property :

$$\left| r_{xy}(l) \right| \leq \sqrt{\varepsilon_x \varepsilon_y}$$

Where $\varepsilon(x)$ and $\varepsilon(y)$ are the energies of the sequences x(n) and y(n) respectively.

(d) The impulse response of a relaxed LTI system is $h(n) = \alpha^n u(n)$, $|\alpha| < 1$. Determine the value of the step response of the system as $n \to \infty$.

(a) Determine the magnitude and phase of H(w) for the system. **10**

$$y(n) = \frac{1}{3}\,[\,x(n+1) + x(n) + x(n-1)\,]$$

and plot these two functions for $0 \leq w \leq \pi$.

(b) A discrete-time system is described by the equation :— **10**

$$y(n) = x(n) - y(n-1) + 6y(n-1)$$

The input to the system is $x(n) = 8u(n)$ with initial conditions $y(-1) = 1$ and $y(-2) = -1$. Determine :

(i) The zero-input response
(ii) The zero-state response
(iii) The total response.

3. (a) Using DFT and IDFT, determine the response y(n) of the FIR filter with impulse response **10**

$h(n) = \{ 1, 2, 3 \}$
$\qquad\uparrow$

to the input sequence

$x(n) = \{ 1, 2, 2, 1 \}$
$\qquad\uparrow$

(b) Given $x(n) = 2^n$ and N = 8, find x(k) using DIT-FFT algorithm. **10**
Verify by any other method.

4. (a) Sketch the block diagram for :
   (i) Direct-form realization
   (ii) Frequency sampling realization

of M = 32, $\alpha = 0$ linear phase (symmetric).
FIR filter which has frequency samples :

$$H\left(\frac{2\pi k}{32}\right) = \begin{cases} 1, & k = 0, 1, 2 \\ \frac{1}{2}, & k = 3 \\ 0 & k = 4, 5.....15 \end{cases}$$

Compare the computational complexity of these two structures.

(b) Obtain the cascade and parallel redisations for the IIR system function given by— 8

$$H(z) + \frac{1+\frac{1}{4}z^{-1}}{\left(1+\frac{1}{2}z^{-1}\right)\left(1+\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}\right)}$$

5. (a) Design a single-pole low pass filter with a 3-dB bandwidth of 0·2 π, by using Bilinear Transformation applied to the analog filter 10

$$H(s) = \frac{\Omega_C}{s+\Omega_C}$$

where Ω_C is the 3-dB bandwidth of the analog filter.

(b) An analog filter has a transfer function : 10

$$H(s) = \frac{10}{s^2+7s+10}$$

Design a digital filter equivalent to this using Impulse Invariant method.

6. (a) A low pass filter is to be designed with the following desired frequency response :— 10

$$H_d\left(e^{jw}\right) = \begin{cases} e^{-j2w}, & -\pi/4 \le w \le \pi/4 \\ 0, & \frac{\pi}{4} < |w| \le \pi \end{cases}$$

Design the filter coefficients $h_d(n)$ if the window function is defined as—

$$w(n) = \begin{cases} 1, & 0 \le n \le 4 \\ 0, & \text{otherwise} \end{cases}$$

Also, determine the frequency response $H(e^{jw})$ of the designed filter.

(b) Consider an analog signal 10

$x(t) = 3 \cos 2000\pi t + 5 \sin 6000\pi t + 10 \cos 12{,}000\pi t$

(i) What is the Nyquist rate for this signal ?
(ii) Assume that we sample the signal using a sampling rate F_s = 5000 samples/sec. What is the discrete-time signal obtained after sampling ?
(iii) What is the analog signal ya(t) we can reconstruct from the samples if we use ideal interpolation ?

7. (a) Compute the inverse Z-transform of the function : 10

$$x(z) = \frac{z}{3z^2-4z+1}$$

For : (i) ROC $|z| > 1$ (ii) ROC $|z| < \frac{1}{3}$ (iii) ROC $\frac{1}{3} < |z| < 1$

(b) Determine the range of values of a and b for which the LTI system with impulse response 5

$$h(n) = \begin{cases} a^n, & n \ge 0 \\ b^n, & n < 0 \end{cases}$$

is stable.

(c) By giving definitions for each, obtain the relationship between DTFT, DFT and Z transform. 5