15109 F.E (Sem II | Rev) All Branches

VT-Mar-09-24(I)

Maths – II

11 am to 2 pm

Con. 2507–09. **(REVISED COURSE)** **VR–1020**

Lib.

(3 Hours) **[Total Marks : 100**

N.B. : (1) Question No. **1** is **compulsory**.

(2) Attempt any **four** questions from remaining **six** questions.

(3) **Figures** to the **right** indicates **full** marks.

1. (a) Prove that $\int_0^1 (x\log x)^4 dx = \frac{4!}{5^5}$ **20**

(b) Evaluate $\iint_R xy(x-1)\,dxdy$, where R is the region bounded by xy = 4, y = 0, x = 1 and x = 4

(c) Find the volume bounded by the cylinder $y^2 = x$, $x^2 = y$ and the planes z = 0 and x + y + z = 2

(d) Solve $\frac{dy}{dx} = \frac{-(x^2y^3+2y)}{(2x-2x^3y^2)}$.

2. Solve the following differential equations – **20**

(a) $\frac{dy}{dx} = \frac{\tan y - 2xy - y}{x^2 - x\tan^2 y + \sec^2 y}$

(b) $\frac{d^2y}{dx^2} + y = \sin x \sin 2x$

(c) $\frac{d^2y}{dx^2} - (a+b)\frac{dy}{dx} + aby = e^{ax} + e^{bx}$

(d) $dr + (2r\cot\theta + \sin 2\theta)d\theta = 0$.

3. (a) Show that $\int_0^a \frac{dx}{\sqrt[n]{(a^n - x^n)}} = \frac{\pi}{n}\operatorname{cosec}\left(\frac{\pi}{n}\right)$, Where n > 1. **6**

(b) Change the order of integration – **6**

$$\int_0^1 \int_{\sqrt{2x-x^2}}^{1+\sqrt{1-x^2}} f(x,y)\,dydx$$

(c) Use the method of variation of parameters to solve — **8**

$$y'' + 3y' + 2y = e^{e^x}$$

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4. (a) Assuming the validity of DUIS, prove that 6

$$\int_0^\infty \left(\frac{e^{-ax} - e^{-bx}}{x}\right) \sin mx\,dx = \tan^{-1}\left(\frac{m}{a}\right) - \tan^{-1}\left(\frac{m}{b}\right).$$

(b) Use spherical polar co-ordinates to evaluate $\iiint xyz\,(x^2 + y^2 + z^2)\,dxdydz$ over the first octant of the sphere $x^2 + y^2 + z^2 = a^2$. 6

(c) Solve using Taylor's series method, the differential equation $\frac{dy}{dx} = x + y$ numerically. Start from x =1, y = 0 and carry to x = 1·2 with h = 0·1. Compare the final result with the value of the exact solution. 8

5. (a) In a single closed circuit, the current 'i' at any time 't' is given by $Ri + L\frac{di}{dt} = E$, find the current i at any time t, given that t = 0, i = 0 and L, R and E are constants. 6

(b) Find the length of the arc of the cardiode $r = a\,(1 - \cos\theta)$, which lies outside the circle $r = a\cos\theta$. 6

(c) Use Euler's modified method to find the values of y satisfying the equation $\frac{dy}{dx} = \log(x+y)$, for x = 1·2 and x = 1·4 correct to three decimals by taking h = 0·2 and y(1) = 2. 8

6. (a) Using Runge-Kutta's fourth order method, find the numerical solution at x = 0·6 for $\frac{dy}{dx} = \sqrt{x + y}$, y(0·4) = 0·41 assume step length, h = 0·2. 6

(b) Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = xe^{-x}\cos x$. 6

(c) Transform to polar co-ordinates and evaluate $\iint \sqrt{\left(\frac{a^2b^2 - b^2x^2 - a^2y^2}{a^2b^2 + b^2x^2 + a^2y^2}\right)}\,dxdy$ where R is the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. 8

7. (a) Find the mass of the leminiscate $r^2 = a^2 \cos 2\theta$, if the density at any point is proportional to the square of its distance from the pole. 6

(b) Solve $(2x+1)^2 \frac{d^2y}{dx^2} - 2\,(2x+1)\frac{dy}{dx} - 12y = 6x$. 6

(c) Sketch the region bounded by the curves $y = x^2$ and $x + y = 2$. Express area of this region as a double integral in two ways and evaluate. 8